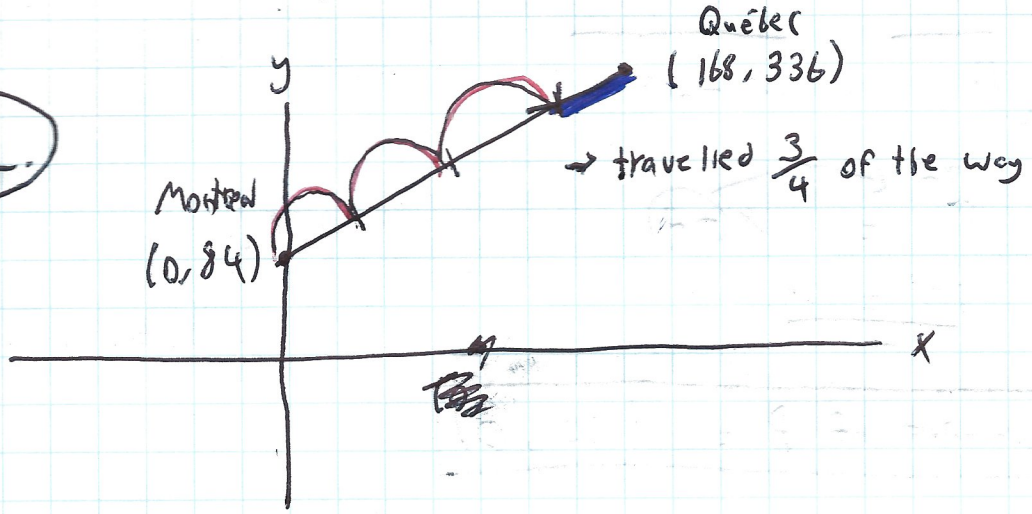
MIDPOINT, DISTANCE and DIVISION

1.



Looking for a point along the line ∴ use division point equation

$$P_{point} = x_1 + \frac{p}{w}(x_2 - x_1), \quad y_1 + \frac{p}{w}(y_2 - y_1)$$

Montreal $(0, 84)$ — x_1, y_1 — Québec $(168, 336)$ — x_2, y_2 — $\frac{p}{w} = \frac{3}{4}$

Starting point

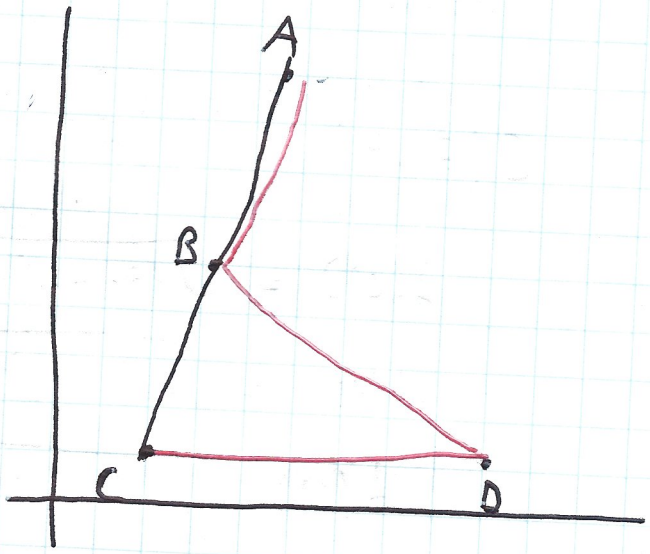
$$P = 0 + \frac{3}{4}(168 - 0), \quad 84 + \frac{3}{4}(336 - 84)$$

$$P = 0 + \frac{3}{4}(168), \quad 84 + \frac{3}{4}(252)$$

$$\boxed{P = (126, 273)}$$

(2.)

A(12,28)
C(2,4)
D(23,4)
B(?,?)



* Get B coordinates by using midpoint formula since B is halfway between A and C.

— = distance traveled by Sabrina

∴ Sabrina travelled a distance from C to D = d(CD)
" " " " " D to B = d(DB)
" " " " " B to A = d(BA)

We need to find each distance, and add them together.

Step 1 find B coordinates → midpoint equation → $x_m = \frac{x_1 + x_2}{2}$

$y_m = \frac{y_1 + y_2}{2}$

$A(\overset{x_1}{12}, \overset{y_1}{28})$ $C(\overset{x_2}{2}, \overset{y_2}{4})$

$x_B = \frac{2 + 12}{2}$, $y_B = \frac{4 + 28}{2}$

$x_B = \frac{14}{2} = 7$, $y_B = \frac{32}{2} = 16$

$\boxed{B(7,16)}$

Step 2 Find distances (let's start with d(CD))

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$C(\overset{x_1}{2}, \overset{y_1}{4})$ $D(\overset{x_2}{23}, \overset{y_2}{4})$

$d(CD) = \sqrt{(23-2)^2 + (4-4)^2}$

$d(CD) = \sqrt{21^2 + 0^2}$

$d(CD) = \sqrt{441 + 0} = \sqrt{441} = \boxed{21 \text{ km}}$

Step 3 Find d(DB)

$D(\overset{x_1}{23}, \overset{y_1}{4})$ $B(\overset{x_2}{7}, \overset{y_2}{16})$

$d(DB) = \sqrt{(7-23)^2 + (16-4)^2}$

$d(DB) = \sqrt{(-16)^2 + (12)^2}$

$d(DB) = \sqrt{256 + 144} = \sqrt{400} = \boxed{20 \text{ km}}$

Step 4 Find d(BA)

$B(\overset{x_1}{7}, \overset{y_1}{16})$ $A(\overset{x_2}{12}, \overset{y_2}{28})$

$d(BA) = \sqrt{(12-7)^2 + (28-16)^2}$

$d(BA) = \sqrt{5^2 + 12^2}$

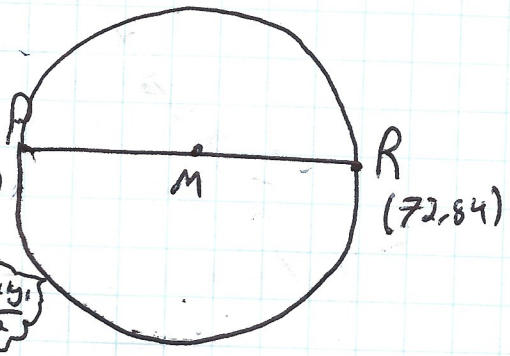
$d(BA) = \sqrt{25 + 144} = \sqrt{169} = \boxed{13 \text{ km}}$

Step 5 Add all distances together

total length of bike ride $= d(CD) + d(BA) + d(DB)$

$21 + 13 + 20 = \boxed{54 \text{ km}}$

3.



Looking for midpoint

↳ $X_m = \frac{x_2+x_1}{2}$, $y_m = \frac{y_2+y_1}{2}$

$P(\overset{x_1}{-12}, \overset{y_1}{36})$ $R(\overset{x_2}{72}, \overset{y_2}{84})$

$X_m = \frac{72 + (-12)}{2}$, $y_m = \frac{84 + 36}{2}$

$X_m = \frac{60}{2}$, $y_m = \frac{120}{2}$

$X_m = 30$, $y_m = 60$

Center of circle = (30, 60)

4.

Looking for point along line.

use division-point formula

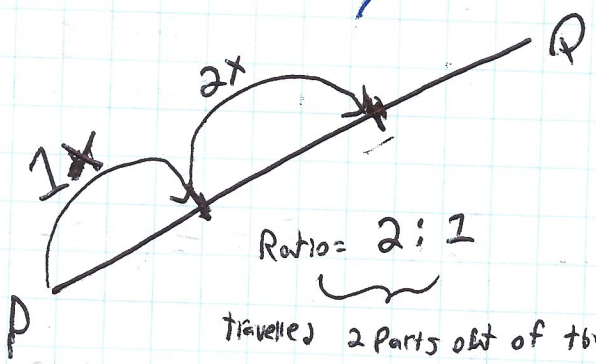
$P_x = x_1 + \frac{P}{W}(x_2 - x_1)$

$P_y = y_1 + \frac{P}{W}(y_2 - y_1)$

$P(0,3)$ $Q(36,18)$
$x_1\ y_1$ $x_2\ y_2$

$P_x = 0 + \frac{2}{3}(36-0) = 24$

$P_y = 3 + \frac{2}{3}(18-3) = 13$



Ratio = 2 : 1

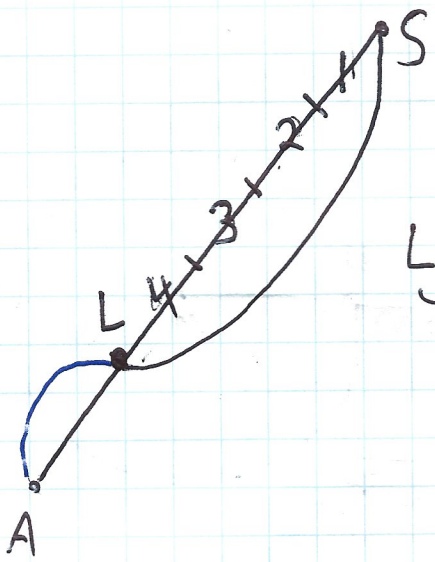
Travelled 2 parts out of the total 3 parts ∴ 2:1 = $\frac{2}{3}$

TRICH

$2:1 = \frac{2}{2+1} = \frac{2}{3}$

Cyclists stop to rest at (24, 13)

5.



L to S = 4x bigger than L to A

4 : 1 ratio

$4:1 = \frac{4}{4+1} = \frac{4}{5}$

$\frac{P}{W}$ in equation starting from S

Looking for point along line so use division-point formula

$$P_x = x_1 + \frac{P}{W}(x_2 - x_1)$$

$$P_y = y_1 + \frac{P}{W}(y_2 - y_1)$$

If travelling from A to L, then you would go $\frac{1}{5}$ of the way so $\frac{P}{W} = \frac{1}{5}$

$A(\overset{x_2}{315}, \overset{y_2}{100})$ $S(\overset{x_1}{915}, \overset{y_1}{900})$

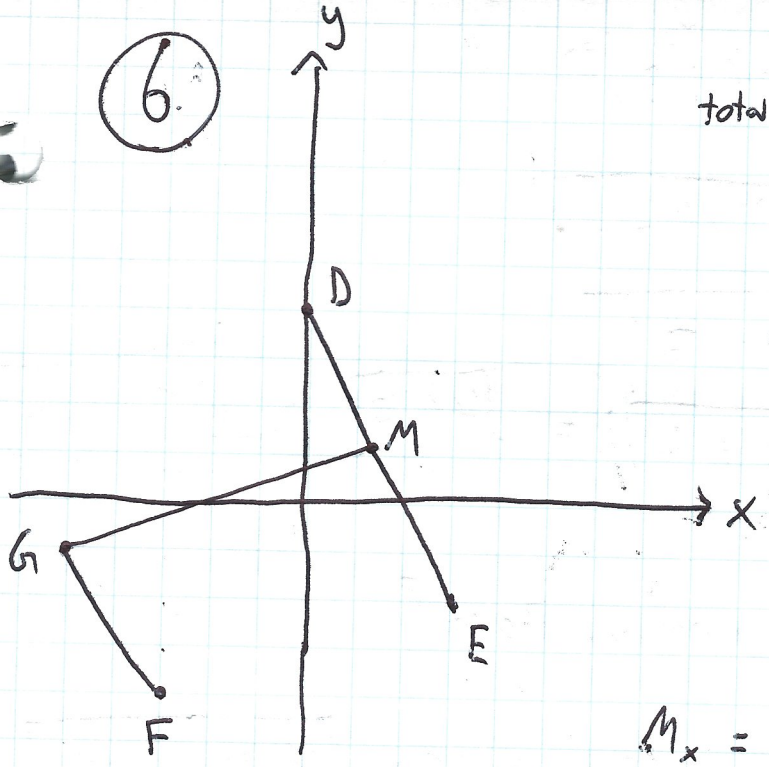
Careful here, starting from point S, so that's why x_1, y_1 is for point S and not point A

$$P_x = 915 + \frac{4}{5}(315 - 915) = 915 + (-480) = 435$$

$$P_y = 900 + \frac{4}{5}(100 - 900) = 900 + (-640) = 260$$

Coordinates of library (L) = (435, 260)

6.



total length of water main = $(\overline{FG} + \overline{GM})$

$F(-3,-4)$

$G(-5,-1)$

$M(?,?)$

need coordinates to get $\overline{GM}$.

∴ use midpoint equation since M is in middle of $\overline{DE}$

$$M_x = \frac{x_1+x_2}{2}$$

$$M_y = \frac{y_1+y_2}{2}$$

Step 1

$D(\overset{x_1}{0},\overset{y_1}{4})$ $E(\overset{x_2}{3},\overset{y_2}{-2})$

$$M_x = \frac{0+3}{2}, \quad M_y = \frac{4+(-2)}{2}$$

$$M_x = \frac{3}{2}, \quad M_y = \frac{2}{2} = 1$$

$$\boxed{M(1.5,1)}$$

Step 2 find distances $\overline{FG}$ and $\overline{GM}$ using distance equation

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$\overline{FG}$

$F(\overset{x_1}{-3},\overset{y_1}{-4})$ $G(\overset{x_2}{-5},\overset{y_2}{-1})$

$$d(FG) = \sqrt{(-5-(-3))^2+(-1-(-4))^2}$$

$$d(FG) = \sqrt{-2^2+3^2} = \sqrt{4+9} = \sqrt{13} = \boxed{3.61 \text{ units}}$$

.6

$\overline{GM}$ $G(\overset{x_1}{-5}, \overset{y_1}{-1})$ $M(\overset{x_2}{1.5}, \overset{y_2}{1})$

$$d(GM) = \sqrt{(1.5-(-5))^2 + (1-(-1))^2}$$

$$d(GM) = \sqrt{6.5^2 + 2^2}$$

$$d(GM) = \sqrt{42.25 + 4} = \sqrt{46.25} = \boxed{6.80 \text{ units}}$$

Step 3

Total length of new water main FGM?

$$\overline{FG} + \overline{GM} = 3.61 + 6.80 = \boxed{10.41 \text{ units}}$$