Cultural, Social and Technical Mathematics
Secondary IV

STUDY GUIDE
This Study Guide has been developed by teachers and consultants with the aim of helping students prepare for the MELS Uniform Examination in Secondary IV CST Mathematics. The production of this guide was possible through funding by an Anglophone community MELS Success Project.

Please note that this document is a “work in progress” and it will be reviewed every school year. Corrections and suggestions should be sent to your school board consultant.

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PREPARING FOR THE EXAM AND EXAM TAKING STRATEGIES

Preparing for the Exam

Preparation is key!

- Pay attention to hints your teacher gives you and take notes.
- Pay regular attention in class and ask for help when needed.
- Go to the tutorial sessions (review).
- Do not leave a topic misunderstood hoping it will not be on the exam. It will very likely be on the exam.
- Budget your time, schedule time to study so that you are well prepared for the test (weeks in advance). Do not wait until the day before!
- Create your own clear and well organized memory aid. This requires planning and time.
- Practice with questions from previous MELS Uniform Exams.
- Complete this booklet.
- Have a good night sleep the night before the exam. Go to bed earlier.
- Have a good breakfast. A healthy meal will give you the mental energy you will need to get through it.

The Day of the Exam

You will need to bring:

- at least two HB pencils and a good eraser
- a calculator (with or without graphic display) but make sure all data and programs are deleted
- a ruler
- your memory aid
- a watch to better pace yourself

Optional:

- a set square, a compass and a protractor
- additional graph paper
Exam Taking Strategies

- Keep a positive attitude and try to stay relaxed.
- When you first receive your test, do a quick read of the entire test in order to appropriately pace yourself. Look for what is easy and what will require more effort.
- Do the easiest problems first.
- Don’t stay on a problem you are stuck on. Come back to it later.
- Read the entire question at least TWICE.
- Watch out for questions with expressions such as: NOT, STRONGEST TO WEAKEST, INCREASING, DECREASING, etc.
- Ask for clarifications, if needed.
- Write legibly and show all your work when required.
- Look over your test (review). Make sure you’ve answered everything.
- Do not leave any blanks.
PREPARING A MEMORY AID

A memory aid consists of one letter size (8 ½” x 11”) sheet of paper IN YOUR HANDWRITING. BOTH sides may be used.

It should contain important information required for the exam. It should be NEAT and ORGANIZED. Have a plan and then write the elements on the sheet of paper. You might have to make more than one memory aid before being satisfied with the results. It’s worth the time and effort.

Make sure to make your OWN memory aid. It is a good way to study for the exam and you will know where to find the items. You may not use someone else’s memory aid (it is considered cheating). And besides, copying someone else’s memory aid may not help you at all.

What can it contain?

- Formulas
- Example problems worked out
- The steps used in the problem listed in order
- Reminders of things to look out for in doing a problem
- Any rules used to solve problems
- Definitions
- Tips and hints

How can you organize it?

- The information should be organized by topic (e.g. triangles: congruent, similar…).
- Use lines (or boxes) to separate the different topics (e.g. a section for analytic geometry where you would include distance between two points, midpoint, slope, etc.).
- Use the resources your teacher suggests
- Make sure you include items you tend to forget.
- Use a color code system or a numbering system.
FORMAT OF THE UNIFORM EXAM

The exam will consist of 3 parts:

A. Multiple Choice Questions (6 questions; 4 marks each)

B. Short Answer Questions (4 questions; 4 marks each)

C. Application Questions (6 questions; 10 marks each)

Part A. Multiple Choice Questions

You will read the questions from a Question Booklet and will choose a statement (A, B, C or D) that best represents your answer. You will answer on a machine-scored answer sheet by filling in a circle using an HB pencil. Make sure you fill it in completely.

In this section, you do not need to show work for marks. Always work out the problem entirely and check all the distractors. Do not stop reading when you think you found the right answer. Read everything. You will be given 4 marks or 0 for each question.

Do not leave a blank! Make a choice even if you don’t know the answer! You have a 25% probability of getting it right.

Part B. Short Answer Questions

You will read the questions from a Question Booklet and will write a statement in the space provided in your Student Booklet.

In this section, you do not need to show work for marks. However, always work out the problem entirely anyways. You will be given 4 marks or 0 for each question. No part marks are given.

Do not leave a blank! Make an educated guess even if you don’t know the answer!
Part C. Application Questions

You will read the questions from your Student Booklet and answer it in the Student Booklet (same booklet).

For each question, you must show all your work to justify your answer. Your work must be organized and clearly presented and cannot simply involve listing the calculator applications used to obtain results or information.

You will be given a mark of 0 if you do not show work or if your work does not justify your answer (even if you have the correct answer).

You will be graded using the evaluation criteria for competency 2:

- **Cr.1** Formulation of a conjecture suited to the situation, if applicable
- **Cr.2** Correct use of appropriate mathematical concepts and processes
- **Cr.3** Proper implementation of mathematical reasoning suited to the situation
- **Cr.4** Proper organization of the steps in an appropriate procedure
- **Cr.5** Correct justification of the steps in an appropriate procedure

The scoring will go as follows. The table on the right is for a conjecture situation. Most problems on the exam are scored according to the table on the left.

<table>
<thead>
<tr>
<th>Observable indicators corresponding to level</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cr. 3</strong></td>
<td>40</td>
<td>32</td>
<td>24</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td><strong>Cr. 2</strong></td>
<td>40</td>
<td>32</td>
<td>24</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td><strong>Cr. 4</strong></td>
<td>20</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td><strong>Cr. 5</strong></td>
<td>20</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

As you can see, you can easily obtain marks for showing some work. Write down your process first (the steps) and then show all your work. At the very least, list the concepts you think apply, write out the applicable formulas, etc. Try something!

**Do not leave a blank!**
SKILL LIST

Can you do the following?

Put a check ✓ in the appropriate box

<table>
<thead>
<tr>
<th>Skill</th>
<th>Yes</th>
<th>Not yet</th>
</tr>
</thead>
<tbody>
<tr>
<td>How to find the distance between two points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the midpoint between two points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the point that divides a line into a given ratio (part to part)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the point that divides a line into a given fraction (part to whole)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the slope of a line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to express an equation in both standard and general form</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the equation of a line given the slope and a point on the line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the equation of a line given two points on the line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the equation of a line parallel to a given line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to find the equation of a line perpendicular to a given line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to determine the position of two straight lines (parallel and perpendicular)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to translate a story into a system of equations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to display a system of relations and their solution on a graph</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to solve a system of equations using the elimination method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to solve a system of equations using the substitution method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>How to solve a system of equations using the comparison method</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

How to determine and interpret the following properties in functions:

- What is a function
- The domain and range of a function
- Where the function is increasing, constant and decreasing
- The minimum and maximum
- The sign of a function
- The y-intercept of a function
- The zeros of a function (x-intercepts)

How to work with the following functions (words, graph, equation, table):

- Zero degree
- First degree (direct)
- First degree (partial- positive and negative slopes)
- 2nd degree (quadratic) function $f(x) = ax^2$
- Exponential function (growth and decay) $f(x) = ac^x$
- Step function
- Periodic function
- Piecewise function
| How to find an **angle** measure using **TRIGONOMETRIC RATIOS (SIN, COS, TAN)** |
| How to find a **side** measure using **TRIGONOMETRIC RATIOS (SIN, COS, TAN)** |
| How to find an **angle or side** measure using **SINE LAW** |
| How to find the **AREA OF A TRIANGLE** - all three methods: |
|  • General formula |
|  • Hero’s formula |
|  • Trigonometric formula |
| How to apply **CLASSIFICATION OF TRIANGLES** |
| How to use **PYTHAGOREAN THEOREM** |
| How to explain the differences in the **properties of QUADRILATERALS** |
| How to find the **areas of triangles/quadrilaterals/regular polygons** |
| How to determine the **angle relationships when parallel lines** are involved |
| How to use **algebra and angle relationships to solve for an unknown (x)** |
| How to **prove that two triangles are congruent** (SSS, SAS and ASA) |
| How to **prove that two triangles are similar** (SSS, SAS and AA) |
| How to find the **unknown side lengths in similar figures** |
| How to find side lengths using **METRIC RELATIONS** |
| How to read a **FREQUENCY TABLE** |
| How to make and read a **STEM AND LEAF PLOT** |
| How to calculate **MEAN DEVIATION** (and what it tells you about the data) |
| How to calculate **PERCENTILE RANK** (and what it means) |
| How to **find a score of place GIVEN PERCENTILE RANK** |
| How to read a **CONTINGENCY TABLE** |
| How to make and **interpret a SCATTER PLOT** |
| How to estimate the **CORRELATION COEFFICIENT** (and what it means) |
| How to determine the **STRENGTH AND DIRECTION** of the **CORRELATION COEFFICIENT** |
| How to determine and represent the **EQUATION OF A REGRESSION LINE** (e.g. Median-Median method, Meyer line method, best fit method) |
| How to **draw a curve** associated with the chosen model |
| How to **interpolate or extrapolate values using a REGRESSION LINE** |
1.1 Points and Segments in the Cartesian Plane
**Question:**

What is the slope of line segment AB?

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

**My Strategies:**

A) $-2$

B) $-\frac{1}{2}$

C) $\frac{1}{2}$

D) 2
**Answer and Solution:**

Find the slope between the two points either with the formula or visually from the graph:

- Formula from a table of values (or just the points):
  
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>90</td>
<td>20</td>
</tr>
</tbody>
</table>

  \[
  \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 20}{90 - 10} = \frac{40}{80} = -\frac{1}{2}
  \]

- Rise over run from the diagram:
  \[
  \frac{\text{rise}}{\text{run}} = \frac{-40}{80} = -\frac{1}{2}
  \]

**Suggested Strategies:**

I) Recognize the table of values is a series of coordinates... you need two sets to find the rule.

II) Choose any coordinate pair and label them \(x_1, y_1\) and \(x_2, y_2\)

III) Find the slope between these coordinate pairs using the formula: \(\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}\)

IV) Plug the slope into the formula \(y = ax + b\)

V) Substitute any of the \((x, y)\) coordinate pairs from the table into the equation and solve for the initial value \((b)\)

VI) If your slope or initial value is in fraction form, multiply each term by the LCM (least common multiple) of the two denominators

VII) Keeping in mind the signs, move all of the terms to one side of the equal sign.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 15 (Slope of a Segment)
Question:
What is the endpoint of a line segment which has one end at (6, 18) and the midpoint at (18, 30)?

A) \((-6, 6)\)  
B) \((12, 24)\)  
C) \((24, 12)\)  
D) \((30, 42)\)

General Strategies:
1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Solve the problem without looking at choices shown (A, B, C and D).  
7. Look at all the choices.  
8. Match your answer to the appropriate choice.  

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

Midpoint formula: 

\[ (18, 30) : \left( \frac{6 + x_2}{2}, \frac{18 + y_2}{2} \right) \]

\[ 18 = \frac{6 + x}{2} \quad 30 = \frac{18 + y}{2} \]

\[ 36 = 6 + x \quad 60 = 18 + y \]

\[ 30 = x \quad 42 = y \]

The answer is (30, 42)

OR

\[ (x, y) \quad x \text{ must be } >18 \quad y \text{ must be } >30 \]

The only answer that meets both conditions is D (30, 42).

A) (-6, 6) This is the answer you get if you mix up the endpoint and the midpoint.

B) (12, 24) This is the answer you get if you just plug the points into the midpoint formula.

C) (24, 12) This is if you just plug in the points into the midpoint formula and mix up the x and y.

D) (30, 42) This is the correct answer.

**Suggested Strategies:**

Use either the mid-point formula, remembering that you will have some algebra to do, or sketch it and see which answer makes sense.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 16 (Point of Division)

Question:

The coordinates of points A and B are A (10, 30) and B (25, 75).

From point B, point P is located \( \frac{3}{5} \) of the way along line segment BA.

Which of the following represents the coordinates of point P?

A) (15.625, 46.875)
B) (16, 48)
C) (19, 57)
D) (19.375, 58.125)

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

Division point formula:

\[
(x_p, y_p) = \left( x_1 + \frac{a}{b} (x_2 - x_1), y_1 + \frac{a}{b} (y_2 - y_1) \right)
\]

With \(a = 3, b = 5, x_1 = 25, y_1 = 75, x_2 = 10, y_2 = 30\)

OR \(a = 2, b = 5, x_1 = 10, y_1 = 30, x_2 = 25, y_2 = 75\)

Example for the x-coordinate with the first choice:

\[
x_p = 25 + \frac{3}{5} (10 - 25)
\]

\[
x_p = 25 + \frac{3}{5} (-15)
\]

\[
x_p = 25 - 9 = 16
\]

Repeat for the y-coordinate.

Here is the sketch:

![Diagram showing points A(10, 30) and B(25, 75) with line segments and ratio points labeled.]

A) Incorrect: this is the answer you get if you use \(\frac{3}{8}\) instead of \(\frac{3}{5}\).

B) Correct

C) Incorrect: this is the answer you get if you calculate the ratio from the wrong end (A to B instead of B to A).

D) Incorrect: this is the answer you get if you use \(\frac{3}{8}\) instead of \(\frac{3}{5}\) and went from the wrong end.

The answer is B.

**Suggested Strategies:**

- Be careful because the problem is stating the distance from B to A, not A to B. You can see that the distractors assume you might make this mistake.

- Determine whether the ratio given is part to part or part to whole. In this case it is part to whole. You can see that the distractors assume you might make the mistake of interpreting it as a part to part ratio.

You can solve the problem by using the distance formula – paying close attention to where you plug in your points. Remember the B to A. You could also use the other part of the ratio and use \(\frac{2}{5}\) of the way from A to B. Don’t let yourself be confused.

You can also sketch the points and see which answer(s) make sense.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 16 (Point of Division)

**Question:**

On a coordinate plane, Jim’s house is situated on a line that runs from his school to the swimming pool.

The school is at point A (200, 800) and the pool is at point B (1200, 1600).

Jim’s house divides line segment AB into a ratio of 4:1 from point A.

What are the coordinates of Jim’s house?

---

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

Jim’s house is situated at ( , ).
Answer and Solution:

Division point formula:

\[(x_p, y_p) = \left( x_1 + \frac{\text{part}}{\text{whole}} (x_2 - x_1), y_1 + \frac{\text{part}}{\text{whole}} (y_2 - y_1) \right)\]

With \( \frac{\text{part}}{\text{whole}} = \frac{4}{4+1} = \frac{4}{5} \)

And \( x_1 = 200, y_1 = 800, x_2 = 1200, y_2 = 1600 \)

\[
x_p = 200 + \frac{4}{5} (1200 - 200)
\]
\[
x_p = 200 + \frac{4}{5} (1000)
\]
\[
x_p = 200 + 800
\]
\[
x_p = 1000
\]

\[
y_p = 800 + \frac{4}{5} (1600 - 800)
\]
\[
y_p = 800 + \frac{4}{5} (800)
\]
\[
y_p = 800 + 640
\]
\[
y_p = 1440
\]

Jim’s house is situated at (1000, 1440).

Suggested Strategies:

Notice the keywords to see what kind of problem it is:
- Line, point
- Divides
- Ratio
- Coordinates

This is a division point question.

Determine whether the ratio given is part to part or part to whole. In this case it is part to part – which means you’ll add the numbers to create the fraction in the formula.

Pay attention to the end from which the ratio is being measured – in this case from point A.

It always helps to make a sketch of the situation.

Additional Resources:

Visions Volume 1, Section 1.1, p. 16 (Point of Division)
**Question:**

How much longer is $\overline{BC}$ than $\overline{AC}$?

$\overline{BC}$ is _________ m longer than $\overline{AC}$.

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer - what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**
**QUESTION 5**

**Answer and Solution:**

\[ d(A, C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

A (-30,40) & C (20, 58)

\[
\begin{align*}
d(A, C) &= \sqrt{(20 - (-30))^2 + (58 - 40)^2} \\
d(A, C) &= \sqrt{50^2 + 18^2} \\
d(A, C) &= \sqrt{2500 + 324} \\
d(A, C) &= \sqrt{2824} \\
d(A, C) &\approx 53.1413 \text{ m}
\end{align*}
\]

B (90,35) & C (20, 58)

\[
\begin{align*}
d(B, C) &= \sqrt{(20 - 90)^2 + (58 - 35)^2} \\
d(B, C) &= \sqrt{(-70)^2 + 23^2} \\
d(B, C) &= \sqrt{4900 + 529} \\
d(B, C) &= \sqrt{5429} \\
d(B, C) &\approx 73.6817 \text{ m}
\end{align*}
\]

\[ 73.6817 - 53.1413 = 20.5404 \text{ m} \]

**Suggested Strategies:**

The key word here is “longer” which implies length. And with the Cartesian plane (coordinates) as part of the question we’ll want to use the distance formula.

Determine the distances we need: AC and BC (we don’t need AB).

And then subtract to find the difference between the two distances calculated.

**Additional Resources:**

Visions Volume 1, Section 1.1, p. 15 (Distance between Two Points)

Question:

Bill says that his house is exactly the same distance to the water tower as Alan’s house is.

Alan does not believe him so he makes a Cartesian plane and puts all the information that he knows is true on the graph.

He starts by making Birch St. the x-axis and Maple Ave. the y-axis since they are perpendicular to each other.

He knows his house is in a straight line with Bill’s and the school is midway on the line between their houses.

He also knows that the water tower is on Maple Ave. 1100 m from Birch St.

Finally he puts the co-ordinates of his house (−400, 200) and the co-ordinates of the school (200, 400) on the graph.

Which of the boys is correct?

General Strategies:

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement. Show any or all your work! Do not leave a blank page!

My Strategies:
Answer and Solution:

Distance between Bill’s house and the water tower:

Bill’s house: (−400, 200)       Water tower: (0, 1100)

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(0 - (-400))^2 + (1100 - 200)^2} \]
\[ d = \sqrt{160000 + 810000} \]
\[ d = \sqrt{970000} \]
\[ d \approx 984.88 \text{ m} \]

Coordinates of Alan’s house:
Bill’s house: (−400, 200)
School: (200, 400), the midpoint

\[ \frac{x_1 + x_2}{2} = x_m \]
\[ \frac{y_1 + y_2}{2} = y_m \]

\[ \frac{-400 + x_2}{2} = 200 \quad \frac{200 + y_2}{2} = 400 \]
\[ -400 + x_2 = 400 \quad 200 + y_2 = 800 \]
\[ x_2 = 800 \quad y_2 = 600 \]

Alan’s house: (800, 600)

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(0 - 800)^2 + (1100 - 600)^2} \]
\[ d = \sqrt{(-800)^2 + (500)^2} \]
\[ d = \sqrt{640000 + 250000} \]
\[ d = \sqrt{890000} \]
\[ d \approx 943.40 \text{ m} \]

Alan is correct; their houses are not the same distance from the water tower.

Suggested Strategies:

Begin by transferring information from the text onto the diagram.

To answer this question you have to calculate the distance between each of the houses and the water tower. For that, you need the three sets of coordinates; Bill’s house, Alan’s house and the water tower.

You are given the coordinates of the water tower (0, 1100) and Bill’s house (−400, 200).

Using the coordinates of Bill’s house and the school, you can determine the coordinates of Alan’s house.

Additional Resources:

Visions Volume 1, Section 1.1, pp. 15-16
1.2 Lines in the Cartesian Plane
Question:

The slope of line l is \( \frac{3}{7} \) and it passes through point (7, 8).

What is the equation of line l?

A) \( y = \frac{3}{7}x + 5 \)

B) \( y = \frac{3}{7}x + 11 \)

C) \( y = \frac{3}{7}x + \frac{25}{7} \)

D) \( y = 5x + \frac{3}{7} \)

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!
Make a choice!

My Strategies:
Answer and Solution:

Formula for a linear equation: \( y = ax + b \)

\[
\text{slope } (a) = \frac{3}{7}
\]

Substitute coordinate values \((7, 8)\) for \(x\) and \(y\) and solve for \(y\)-intercept \(b\)

\[
y = \frac{3}{7}x + b
\]

\[
8 = \frac{3}{7}(7) + b
\]

\[
8 = 3 + b
\]

\[
8 - 3 = b
\]

\[
5 = b
\]

\[
y = \frac{3}{7}x + 5
\]

The answer is A.

Suggested Strategies:

I) Plug the slope into the formula \( y = ax + b \)

II) Substitute the \((x, y)\) coordinate pair into the equation and solve for the initial value or \(y\)-intercept \((b)\)

III) Keeping in mind the signs, move all of the terms to one side of the equal sign.

IV) Re-write the formula with the slope and initial value to yield the equation of the line

Additional Resources:

Visions Volume 1, Section 1.2, p. 26 (Equation of Line from Slope and Intercepts)


Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:

- Point-Slope Form of a Line
- Slope-Intercept Form of a Line
Question:

Which of the following equations represents a line perpendicular to $4x + 3y + 12 = 0$?

A) $y = -\frac{3}{4}x + 2$
B) $y = \frac{4}{3}x - 4$
C) $y = \frac{3}{4}x - 2$
D) $y = -\frac{4}{3}x + 4$

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

Find the slope of \(4x + 3y + 12 = 0\)

\[
3y = -4x - 12 \\
\frac{3y}{3} = \frac{-4x - 12}{3} \\
y = \frac{-4}{3}x - 4 \quad \text{the slope is} \quad \frac{-4}{3}
\]

Find the perpendicular slope: \(\frac{a}{b} \rightarrow -\frac{b}{a}\)

The perpendicular slope is \(\frac{3}{4}\) (options B and D are ‘out’)

Option A

\[
y = -\frac{3}{4}x + 2
\]

The slope is not \(\frac{3}{4}\), option A is wrong.

Option C

\[
y = \frac{3}{4}x - 2
\]

The slope is equal to \(\frac{3}{4}\), option C is correct.

The answer is C.

Suggested Strategies:

I) The word perpendicular in this problem should immediately cause you to write the negative reciprocal rule for perpendicular slopes: \(\frac{a}{b} \rightarrow -\frac{b}{a}\)

II) Start by converting the equation from ‘general’ form to ‘slope-intercept’ form in order to get a better look at the slope.

III) Find the negative reciprocal of the slope from the equation given in the problem. This is the slope we are looking for in our multiple-choice answers.

IV) Remember that we only care about finding a perpendicular line in this problem, so we only need to worry about the slopes. Ignore the initial values altogether... they are only distractors here.

Additional Resources:

Visions Volume 1, Section 1.2, p. 27 (Perpendicular Line)
Question:

What is the x-intercept for the following linear equation:

\[ 2x + 3y + 6 = 0 \]

A) \(-3\)

B) \(-2\)

C) 2

D) 3

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Question 9

Answer and Solution:

Set the ‘y’ value to 0 and solve for x.

\[ 2x + 0 + 6 = 0 \]
\[ 2x + 6 = 0 \]
\[ 2x = -6 \]
\[ x = \frac{-6}{2} \]
\[ x = -3 \]

The answer is A.

Suggested Strategies:

I) Remember that the x-intercept is the point on a graph where the line crosses the x-axis \((y = 0)\)

II) Set \(y = 0\) and solve for \(x\).

Additional Resources:

Visions Volume 1, Section 1.2, p. 26 (Equation of a Line)
Question:

What is the equation of the line parallel to $3x - 4y - 24 = 0$ that passes through point $P(-8, 7)$?

A) $y = \frac{3}{4}x + \frac{13}{4}$

B) $y = \frac{3}{4}x + 13$

C) $y = \frac{3}{4}x + 52$

D) $y = -\frac{4}{3}x - \frac{13}{3}$

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem **without looking** at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

Convert the equation from general form to $y$-intercept form

\[
3x - 4y - 24 = 0
\]

\[
-4y = -3x + 24
\]

\[
\frac{-4y}{-4} = \frac{-3x + 24}{-4}
\]

\[
y = \frac{3x}{4} - 6 \text{, the slope of the parallel line must be } \frac{3}{4}
\]

\[
y = ax + b
\]

\[
y = \frac{3}{4}x + b
\]

\[
7 = 0.75(-8) + b
\]

\[
7 = -6 + b
\]

\[
13 = b
\]

\[
y = \frac{3}{4}x + 13
\]

The answer is B.

**Suggested Strategies:**

I) Remember that parallel lines always have the same slope.

II) Convert the rule in the question from ‘general’ form to ‘slope-intercept’ form in order to find the slope. This slope will be the same in your new parallel line.

III) Plug the parallel slope (a) into the formula $y = ax + b$

IV) Substitute the coordinates of point $P (-8, 7)$ into the new equation and solve for the initial value (b).

V) Additional Resources:

Visions Volume 1, Section 1.2, p. 27 (Parallel Line)

Question:

Line $L_1$ passes $P (12, 15)$ and is perpendicular to line $L_2$
Line $L_2$ has equation $-4x + 5y - 10 = 0$.

What is the $x$-intercept of the line $L_1$?

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer-what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.

My Strategies:

The $x$-intercept of the line $L_1$ is ____________
Answer and Solution:
Convert the equation line L₂ from general form to standard form
\[-4x + 5y - 10 = 0\]
\[5y = 4x + 10\]
\[5y = 4x + 10\]
\[5 = \frac{4x}{5}\]

\[y = \frac{4x}{5} + 2, \text{ the slope of the line } L₂ \text{ is } \frac{4}{5}\]

Find the perpendicular slope of line L₁: \(\frac{a}{b} \rightarrow -\frac{b}{a}\), \(\frac{4}{5} \rightarrow \frac{-5}{4}\)
\[y = ax + b\]
\[y = -\frac{5}{4} x + b\]

\[15 = -1.2(12) + b\]
\[15 = -14.4 + b\]
\[30 = b\]
\[y = -\frac{5}{4} x + 30\]

Set the ‘y’ value to 0 and solve for x.

\[0 = -\frac{5}{4} x + 30\]
\[-30 = -\frac{5}{4} x\]
\[-30 \left(\frac{4}{-5}\right) = x\]
\[24 = x\]

The x-intercept of the line L₁ is 24

Suggested Strategies:
I) Remember that an x-intercept is the point at which a line crosses the x-axis. The y-value of this coordinate must be equal to 0 (y = 0) at this point.

II) Start by converting the equation from general form to standard form in order to get a better look at the slope.

III) The word perpendicular in this problem should immediately cause you to write the negative reciprocal rule for perpendicular slopes: \(\frac{a}{b} \rightarrow -\frac{b}{a}\).

IV) Since we are looking for the line that is perpendicular to \(-4x + 5y - 10\), we’ll need the negative reciprocal \((-\frac{b}{a})\) of the slope from the equation given in the problem.

V) Use the perpendicular slope in a new ‘y = ax + b’ rule.

VI) Substitute the coordinates of point P (12, 15) into the new equation and solve for the initial value (b).

VII) Once you’ve got your rule for the perpendicular line finished, find the x-intercept by making y = 0 and solving for x.

Additional Resources:
Visions Volume 1, Section 1.2, pp. 26-27
Question:

Line $L_1$ has a slope of $\frac{4}{3}$ and a $y$-intercept of $-3$.

Line $L_2$ is perpendicular to line $L_1$ and passes through point $(2, 5)$.

What is the equation of line $L_2$?

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:

The equation of line $L_2$ is _______________________________
**Answer and Solution:**

Line $L_1$:

\[ y = \frac{4x - 3}{3} \]

Line $L_2$:

\[ a = \frac{-3}{4} \]

Substitute point $(2, 5)$ in for the $x$ and $y$ to solve for “$b$”.

\[ y = \frac{-3x}{4} + b \]

\[ y = \frac{-3(2)}{4} + b \]

\[ 5 = \frac{-6}{4} + b \]

\[ b = 5 + \frac{3}{2} = 6.5 \]

\[ y = \frac{-3x}{4} + 6.5 \]

The equation of line $L_2$ is $y = \frac{-3x}{4} + 6.5$

**Suggested Strategies:**

- Use the negative reciprocal of the slope of line $L_1$ to find the slope of line $L_2$.

- Use the function form of the equation, $y = ax + b$, with point $(2, 5)$ and the new slope.

**Additional Resources:**

Visions Volume 1, p. 27 (Mathematical Knowledge Summary)
Question:

Given the equation:

\[ 8x + 6y + 12 = 0 \]

A) What is the \( x \)-intercept?

B) What is the \( y \)-intercept?

The \( x \)-intercept is ____________

The \( y \)-intercept is ____________

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

Algebraically:

A) x-intercept

\[ y = 0 \]
\[ 8x + 6y + 12 = 0 \]
\[ 8x + 6(0) + 12 = 0 \]
\[ 8x = -12 \]
\[ x = \frac{-12}{8} = \frac{-3}{2} \]

B) y-intercept

\[ x = 0 \]
\[ 8(0) + 6y + 12 = 0 \]
\[ 6y + 12 = 0 \]
\[ 6y = -12 \]
\[ y = \frac{-12}{6} = -2 \]

Graphically:

\[ 8x + 6y + 12 = 0 \rightarrow y = -\frac{4}{3}x - 2 \]

The x-intercept is \( \frac{-3}{2} \).

The y-intercept is \(-2\).

Suggested Strategies:

When finding the intercepts, the other coordinate is 0:

x-intercept means \( y = 0 \)

y-intercept means \( x = 0 \).

You can also solve this question graphically by changing the equation into function form and plotting the y-intercept and slope.

Additional Resources:

Visions Volume 1, p. 26 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelarning.com/ look up:
Question:

A car is travelling along a straight path from point A (−24, −39) to point B (30, 33).

The car breaks down, having completed exactly two-thirds of the trip.

A tow truck must travel along a path that is perpendicular to the car’s path, leaving from a garage located somewhere along the x-axis.

How far must the tow-truck travel to get from the garage to the car? (All units are in km.)

General Strategies:

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

My Strategies:
Answer and Solution:

Find the coordinates of the point where the car breaks down

Division point 2/3 of the way between A and B

\[ \text{Division point: } \left( x_1 + \frac{a}{b} (x_2 - x_1), y_1 + \frac{a}{b} (y_2 - y_1) \right) \]

A \((-24, -39)\)
B \((30, 33)\)

\[ \left( -24 + \frac{2}{3} (30 - (-24)), -39 + \frac{2}{3} (33 - (-39)) \right) \]
\[ \left( -24 + \frac{2}{3} (54), -39 + \frac{2}{3} (72) \right) \]

Call the position of the car point C

C \((12, 9)\)

Find the slope of the rule for the line the car travels

**Formula for a linear equation:** \( y = ax + b \)

**Coordinates used to find the rule:**

A \((-24, -39)\)
B \((30, 33)\)

\[ \text{slope (a)} = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ \text{slope (a)} = \frac{33 - (-39)}{30 - (-24)} = \frac{72}{54} = \frac{4}{3} \]

Find the equation for the tow-truck’s path

The tow-truck’s path will be perpendicular (negative reciprocal slope) and passes through point C \((12, 9)\), the car

\[ \frac{a}{b} \rightarrow \frac{-b}{a}, \quad \frac{4}{3} \rightarrow -\frac{3}{4}, \quad \text{the tow-truck’s slope is} \quad -\frac{3}{4} \]

\[ y = ax + b, \quad \text{passing through} \quad (12, 9) \]

\[ y = -\frac{3}{4}x + b \]
\[ 9 = -\frac{3}{4} (12) + b \]
\[ 9 = -9 + b \]
\[ 18 = b \]

Suggested Strategies:

I) Recognize that the question is asking for a distance between two points, the car and the garage. This problem requires us to first find and then use those coordinates.

II) Start by using the division point formula to find the coordinates of the car when it breaks down.

III) Then find the coordinates of the garage,

   a. we know that it is on the x-axis (so the y-coordinate is 0)

   b. We know that it is on the path that is perpendicular to AB

IV) Find the slope of AB so we can use its negative reciprocal to define the slope of the line between the car and the garage.

V) Plug the \((x, y)\) coordinates of the car into the formula for the tow-truck’s path, then solve for the initial value to complete the equation for the tow-truck’s path.
\[ y = \frac{-3}{4}x + 18 \]

Find the coordinates of the Garage on the x-axis

Set the ‘y’ value to 0 and solve for x.

\[ y = \frac{-3}{4}x + 18 \]
\[ 0 = \frac{-3}{4}x + 18 \]
\[ -18 = \frac{-3}{4} \]
\[ -18 \left( \frac{4}{-3} \right) = x \]
\[ 24 = x \]

The coordinates of the garage are: (24, 0)

Find the distance from the garage to the car.

Garage (24, 0)
Car (12, 9)

\[
distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
distance = \sqrt{(12 - 24)^2 + (9 - 0)^2} \\
distance = \sqrt{(-12)^2 + (9)^2} \\
distance = \sqrt{144 + 81} \\
distance = 15
\]

Final answer: the distance the tow-truck must travel from the garage to the car is 15 km.

**Additional Resources:**

Visions Volume 1, Section 1.2, pp. 26-27
Visions Volume 1, Section 1.1, pp. 15-16
1.3 Systems of Equations
Question:

Consider the following system of linear equations.

\[ y = -\frac{2}{3}x - 2 \]

\[ y = -\frac{2}{3}x - 4 \]

Which of the following statements is true?

A) The system has an infinite number of solutions.
B) The system has a unique solution.
C) The system has no solution.
D) The system has two solutions.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

The slopes \(a\) are the same and the \(y\)-intercepts \(b\) are different; if graphed you would see that the lines are parallel and never intersect.

The answer is C.

**Suggested Strategies:**

Solve the system.

Check to see if the slopes are the same;
- If not, there will be one solution;
- If the slopes are the same, check to see if the \(y\)-intercepts are the same
  - If they are the same, there is an infinite number of solutions since they are the same line;
  - If they are not, they are parallel lines and the system has no solution.

**Additional Resources:**

Visions Volume 1, p. 40 (Mathematical Knowledge Summary)
Question:

Given the following system of equations.

\[2x - 5y + 12 = 0\]
\[x - 3y = 4\]

What is the solution for this system?

A) \((-41, -14)\)

B) \((-44, -20)\)

C) \((-56, -20)\)

D) \((-56, -12)\)

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

\[2x - 5y + 12 = 0\]
\[x - 3y = 4\]

Substitution method (since it is easy to isolate \(x\).)
\[x - 3y = 4\]
\[x = 3y + 4\]
\[2x - 5y + 12 = 0\]
\[2(3y + 4) - 5y + 12 = 0\]
\[6y + 8 - 5y + 12 = 0\]
\[y + 20 = 0\]
\[y = -20\]
\[x = 4 + 3y\]
\[x = 4 + 3(-20)\]
\[x = 4 - 60\]
\[x = -56\]
\[(-56, -20)\]

Check:
\[2x - 5y + 12 = 0\]
\[2(-56) - 5(-20) + 12 = 0\]
\[-112 + 100 + 12 = 0\] True
\[x - 3y = 4\]
\[(-56) - 3(-20) = 4\]
\[-56 + 60 = 4\] True

Both are true, so \((-56, -20)\) is the correct solution.

**The answer is C.**

**Suggested Strategies:**

**Method 1:**
Solve the system of equations by the method of your choice – this one lends itself to substitution.

**Method 2:**
Check by substituting each possible answer into the two equations to verify which point is a possible solution.

\((-41, -14)\)
\[2x - 5y + 12 = 0\]
\[2(-41) - 5(-14) + 12 = 0\]
\[-82 + 70 + 12 = 0\]
\[0 = 0\]
\[x - 3y = 4\]
\[-41 - 3(-14) = 4\]
\[-41 + 52 = 4\]
\[11 \neq 4\]

\((-44, -20)\)
\[2x - 5y + 12 = 0\]
\[2(-44) - 5(-20) + 12 = 0\]
\[-88 + 100 + 12 = 0\]
\[24 \neq 0\]

\((-56, -12)\)
\[2x - 5y + 12 = 0\]
\[2(-56) - 5(-12) + 12 = 0\]
\[-112 + 60 + 12 = 0\]
\[-40 \neq 0\]

**Additional Resources:**

Visions Volume 1, p. 39 (Mathematical Knowledge Summary)
**Question:**

Stephane purchased 3 chocolate chip cookies and 4 peanut butter cookies for $5.65. Marie purchased 5 chocolate chip cookies and 7 peanut butter cookies for $9.70.

What is the price of a peanut butter cookie?

A) 70 cents  
B) 75 cents  
C) 80 cents  
D) 85 cents

**General Strategies:**

1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Solve the problem without looking at choices shown (A, B, C and D).  
7. Look at all the choices.  
8. Match your answer to the appropriate choice.  

Do not leave a blank! Make a choice!

**My Strategies:**
**Answer and Solution:**

\[ x = \text{cost of a chocolate chip cookie} \]
\[ y = \text{cost of a peanut butter cookie} \]

\[
\begin{align*}
3x + 4y &= 5.65 \\
5x + 7y &= 9.70
\end{align*}
\]

\[
-5(3x + 4y = 5.65) \\
3(5x + 7y = 9.70)
\]

\[
\begin{align*}
-15x - 20y &= -28.25 \\
15x + 21y &= 29.10
\end{align*}
\]

\[ y = 0.85 \]

Cost of a peanut butter cookie = $0.85

A) 70 cents  
B) 75 cents – is cost of chocolate chip cookies  
C) 80 cents  
D) 85 cents

The answer is D.

**Additional Resources:**

Visions Volume 1, pp. 39-40 (Mathematical Knowledge Summary)  

**Suggested Strategies:**

Set up a system of equations and solve it.

- Define your variables
- Write your equations
- Choose a method (this one suggests elimination method but the other methods work as well.)
- Interpret your answer correctly by seeing which variable represents the cost of the peanut butter cookie.
Question:

A kitchen cabinetmaker has two models of upper cupboards which a client can choose from to complete a kitchen installation.

Three different clients ordered different combinations of tall and short cabinets. The total cost including delivery is listed below for clients A and B.

<table>
<thead>
<tr>
<th>Client</th>
<th>Number of Tall Cabinets</th>
<th>Number of Short Cabinets</th>
<th>Delivery Cost</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7</td>
<td>4</td>
<td>$120</td>
<td>$1840</td>
</tr>
<tr>
<td>B</td>
<td>9</td>
<td>8</td>
<td>$190</td>
<td>$2630</td>
</tr>
<tr>
<td>C</td>
<td>11</td>
<td>2</td>
<td>$170</td>
<td>?</td>
</tr>
</tbody>
</table>

Client C believes his total cost is lower than client B’s.

Is he correct?

☐ Yes

☐ No
Answer and Solution:

\[ x = \text{cost for long cabinet} \]
\[ y = \text{cost for short cabinet} \]

Client A: \[ 7x + 4y + 120 = 1840 \]
OR \[ 7x + 4y = 1720 \]
Client B: \[ 9x + 8y + 190 = 2630 \]
OR \[ 9x + 8y = 2440 \]
Client C: \[ 11x + 2y + 170 = ? \]

By elimination method:

\[
\begin{align*}
\text{Step 1) } & \quad 7x + 4y = 1720 \\
\text{Step 2) } & \quad 9x + 8y = 2440 \\
\end{align*}
\]

\[
\begin{align*}
9x + 8y & = 2440 \\
-2(7x + 4y) & = -2(1720) \\
7x + 4y & = 1720 \\
13x + 0 & = -440 \\
13x & = -440 \\
x & = -34 \\
\end{align*}
\]

\[
\begin{align*}
11x + 2y & = 170 \\
-2(200) & = -400 \\
8y & = 170 - 400 \\
8y & = -230 \\
y & = -28.75 \\
\end{align*}
\]

\[
\begin{align*}
x & = -34 \\
y & = -28.75 \\
\end{align*}
\]

\[
\begin{align*}
x & = 200 \\
y & = 80 \\
\end{align*}
\]

Client C: \[ 11x + 2y + 170 = ? \]
\[ 11(200) + 2(80) + 170 = 2200 + 160 + 170 = 2530 \]

$2530

Client C is correct. His total cost will be lower than client B’s, since it is $2530 compared to $2630.

Suggested Strategies:

This is a “system of equations” question.

In order to find the cost for Client C, you need to know how much each type of cabinet costs.

Use the information given for the other two clients to find those costs.

- Define your variables,
- Set up two equations in two unknowns,
- Solve the system,
- Use the solution to find the cost for Client C

Note: if you don’t show any work and just check one of the boxes, you will get zero.

Additional Resources:

Visions Volume 1, pp. 39-40 (Mathematical Knowledge Summary)
2.1 Diagrams and Statistics (Dispersion, Deviation, Stem and Leaf...)
Question:

What is the mean deviation of the following set of data?

21 21 21 23 23 23

A) 0  
B) 1  
C) 2  
D) 6  

General Strategies:

1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Solve the problem without looking at choices shown (A, B, C and D).  
7. Look at all the choices.  
8. Match your answer to the appropriate choice.  
Do not leave a blank!  
Make a choice!  

My Strategies:
**Answer and Solution:**

1. Mean = \( \frac{21+21+21+23+23+23}{6} \)  
   Mean = 22

2. |21-22| = |-1| = 1  
   |21-22| = |-1| = 1  
   |21-22| = |-1| = 1  
   |23-22| = 1  
   |23-22| = 1  
   |23-22| = 1

3. Mean deviation = \( \frac{1+1+1+1+1+1}{6} \)  
   Mean deviation = 1

   A) 0 – if you didn’t take the absolute value of the differences

   B) 1 – correct

   C) 2 – unlikely, but just in case you take the difference between the two repeated values

   D) 6 – if you forget to divide by 6

The answer is B.

**Specific Strategies:**

1. Calculate the mean of the set of data.
2. Subtract the mean from each value in the set of data and determine its absolute value. (Remember absolute values can’t be negative.)
3. Calculate the mean of the deviations.

**Additional Resources:**

Visions Volume 1, p. 81  
[http://www.wikihow.com/Calculate-Mean-Deviation-About-Mean-(for-Ungrouped-Data)]
Question:

Which of the following statements is/are true concerning statistical measures?

I. The mean, median and range are measures of central tendency.

II. Percentile rank is a measure of dispersion.

III. The mean deviation and range are measures of dispersion.

IV. The mean deviation is a measure of position.

A) I only
B) III only
C) II and III only
D) II and IV only

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

I. The mean, median, and range are measures of central tendency. False – range is a measure of dispersion.

II. Percentile rank is a measure of dispersion. False – percentile is a measure of position.

III. **The mean deviation and range are measures of dispersion.** True.

IV. The mean deviation is a measure of position. False – mean deviation is a measure of dispersion.

**The answer is B.**

**Suggested Strategies:**

It is important to remember your vocabulary. Instead of blindly calculating mean, range etc. Try to think about why you are doing them and what the result represents. This goes for any stats question.

Recall:
- The measures of central tendency are mean, median, and mode.
- The measures of position are percentile rank
- Range and mean deviation are measures of dispersion.

**Additional Resources:**

Visions Volume 2, pp. 81-82
Question:

Consider the stem-leaf plot below showing the number of sit-ups students do in 60 seconds.

<table>
<thead>
<tr>
<th>Number of Sit-ups</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>0 1 1 2 2 8 9</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>2 2 3 4 5 6 6 8 9</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>1 1 2 3 4 4 5 6 7 8</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>0 1 1 2 5 6 6 7 8 8 8</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>2 4 6 6</td>
</tr>
</tbody>
</table>

How many sit-ups did a student do if he is ranked in the 70th percentile?

A) 35
B) 36
C) 51
D) 52

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

1. There are 42 values in the stem and leaf plot

2. Apply the formula: \( \frac{\text{percentile}}{100} \times \text{number of values} \)

\[
\frac{70}{100} \times 42 = 0.70 \times 42 = 29.4
\]

3. Always round down the result (round to 29).
4. The answer represents the POSITION from the bottom (lowest result) of the data value you wish to find.

<table>
<thead>
<tr>
<th>Number of sit-ups</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>0 1 1 2 2 8 9</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>2 2 3 4 5 6 6 8 9</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>1 1 2 3 4 4 5 6 7 8</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>0 1 2 2 5 6 6 7 8 8 8</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>2 4 6 6</td>
</tr>
</tbody>
</table>

A) This is the result when you make both mistakes as described in B) and D).
B) This is the result when you count from the wrong end.
C) **This is correct.**
D) This is the result when you round up instead of down – it’s the 30th position.

The answer is C.

Specific Strategies:

- Read the stem and leaf plot correctly.
- Do the calculations completely before selecting your answer.

The percentile is given which means you need to work backwards to find the data value.

1. Find the total amount of values
2. Find the position from the bottom by applying the formula
3. Locate that position according to value found in step 3.

Additional Resources:


Question:

At a large company, a survey was conducted to see how fast employees can type. The company has 305 employees. The partial list below shows the speed, in words per minute, achieved by the employees:

30, 32, 32, ... 49, 50, 50, 50, 52, 53, 53, ... 89, 90, 93, 99

| 146 employees | 3 employees | 156 employees |

What is the percentile rank for an employee who types 50 words per minute?

A) 47
B) 48
C) 49
D) 50

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

Answer using the formula:

\[
\frac{\text{number of data values below } x + \frac{\text{number of data values equal to } x}{2}}{\text{total number of data values}} \times 100
\]

\[
\frac{146 + \frac{3}{2}}{305} \times 100 = \frac{146 + 1.5}{305} \times 100 = \frac{147.5}{305} \times 100
\]

\[
\approx 48.36 \text{ round up to 49}
\]

A) 47 – if you don’t take into account the 3 values at 50

B) 48 – if you round down or don’t take into account the 3 values at 50

C) 49 – correct

D) 50 – if you just take the value itself

The answer is C.

Suggested Strategies:

The question is asking for percentile, so you need the formula which gives you the percentile of a data value.

Be careful:

Do not use the formula for finding a data value when the percentile is given!

Remember to round to the next whole number (always UP!)

Additional Resources:

Visions Volume 1, Section 2.1, pp. 76-88
Question:
Consider the following set of data:

\[41 \ 17 \ 25 \ 9 \ 20 \ 12 \ 11 \ 21 \ 20\]

What is the mean deviation for the set of data?

The mean deviation for the set of data is ______.

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

It is suggested to organize your work in a table.

1. The mean of the set of data is 19.55.
2. See the third column of the table.

| Value | Mean  | |Difference of value from mean|
|-------|-------|-----------------------------|
| 41    | 19.55 | 21.45                       |
| 17    | 19.55 | 2.55                        |
| 25    | 19.55 | 5.45                        |
| 9     | 19.55 | 10.55                       |
| 20    | 19.55 | 0.45                        |
| 12    | 19.55 | 7.55                        |
| 11    | 19.55 | 8.55                        |
| 21    | 19.55 | 1.45                        |
| 20    | 19.55 | 0.45                        |

Total: 58.45

3. The mean deviation is:

\[ \text{Mean deviation} = \frac{58.45}{9} = 6.49 \]

The mean deviation for the set of data is 6.49.

Specific Strategies:

1. Calculate the mean of the set of data
2. Subtract the mean from each value in the set of data
3. Calculate the mean of the deviations

Additional Resources:

Visions Volume 1, p. 81
http://www.mathsisfun.com/data/mean-deviation.html

Remember to take the absolute value of each deviation NO NEGATIVE VALUES!
The table below shows the finishing times for the 137 runners participating in a 5 km race:

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>18:48</td>
<td>26:36</td>
<td>29:22</td>
<td>31:34</td>
<td>35:08</td>
<td>38:04</td>
</tr>
<tr>
<td></td>
<td>20:01</td>
<td>26:37</td>
<td>29:29</td>
<td>31:55</td>
<td>35:09</td>
<td>38:45</td>
</tr>
<tr>
<td></td>
<td>24:29</td>
<td>27:50</td>
<td>29:56</td>
<td>32:50</td>
<td>36:11</td>
<td>41:07</td>
</tr>
<tr>
<td></td>
<td>24:34</td>
<td>27:50</td>
<td>30:02</td>
<td>32:56</td>
<td>36:12</td>
<td>41:35</td>
</tr>
<tr>
<td></td>
<td>24:34</td>
<td>27:56</td>
<td>30:03</td>
<td>32:57</td>
<td>36:24</td>
<td>41:35</td>
</tr>
<tr>
<td></td>
<td>25:01</td>
<td>28:42</td>
<td>30:28</td>
<td>33:09</td>
<td>36:25</td>
<td>44:45</td>
</tr>
<tr>
<td></td>
<td>25:58</td>
<td>29:04</td>
<td>31:07</td>
<td>33:46</td>
<td>37:43</td>
<td>47:05</td>
</tr>
<tr>
<td></td>
<td>26:19</td>
<td>29:13</td>
<td>31:25</td>
<td>33:46</td>
<td>37:54</td>
<td>47:19</td>
</tr>
<tr>
<td></td>
<td>26:24</td>
<td>29:17</td>
<td>31:27</td>
<td>34:22</td>
<td>37:58</td>
<td>47:19</td>
</tr>
<tr>
<td></td>
<td>26:31</td>
<td>29:17</td>
<td>31:29</td>
<td>34:43</td>
<td>38:03</td>
<td>48:11</td>
</tr>
</tbody>
</table>

A) What is the percentile rank of a runner with a finishing time of 28 minutes 45 seconds?

B) What is the finishing time of the runner who ranked in the 60th percentile?

The percentile rank of the runner finishing with 28:45 is _______.

The finishing time of the runner ranked in the 60th percentile is _______.

My Strategies:
Answer and Solution:

Solution A

Percentile rank =

\[
\frac{\text{total number of values below}}{\text{total number of values}} + \frac{\text{total number of values equal to}}{2} \times 100
\]

\[
\frac{103 + \frac{2}{2}}{137} \times 100 = 75.9
\]

Round up to the next integer: 76th

Solution B

Percentile rank =

\[
\frac{\text{percentile rank}}{100} \times \# \text{ of values} = \text{position}
\]

\[
\frac{60}{100} \times 137 = 82.2 \text{ or 82 from the bottom}
\]

or

\[
137 - 82 = 55 \text{ runners finished ahead of him}
\]

He is the 56th runner from the top/82nd from the bottom with a time of 30:39.

Note: Your answer might be different if you use a different percentile formula.

A) The percentile rank of a runner finishing with 28:45 is the 76th.

B) The finishing time of a runner ranked in the 60th percentile is 30:39.

Additional Resources:

Visions Volume 1, p. 81-82
Question:

The 20 best swimmers from across the country are trying out for the national swim team. To earn a spot on the team, a swimmer must meet both the following criteria:

**Criteria 1**
The swimmer must rank better than the 60th percentile.

**Criteria 2**
The swimmer must have a “personal best time” (PBT) that is less than or equal to 20 seconds minus the mean deviation (MD) of the group.

\[ \text{PBT} \leq 20 - \text{MD} \]

<table>
<thead>
<tr>
<th>“Personal Best Times” (in seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.56</td>
</tr>
<tr>
<td>18.7</td>
</tr>
<tr>
<td>18.9</td>
</tr>
<tr>
<td>18.95</td>
</tr>
<tr>
<td>19.2</td>
</tr>
</tbody>
</table>

The mean of this distribution is 19.77 seconds.

How many of the 20 swimmers will earn a spot on the National team?

**General Strategies:**
1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

**My Strategies:**
Answer and Solution:

Criteria 1:
Swimmers that meet qualification 1:

\[
\frac{60}{100} \times 20 = 12
\]

This means 12 swimmers are at or below the 60th percentile.

If 12 of the 20 swimmers are at or below the 60th percentile, then 8 are above it.

Criteria 2:
Mean of the distribution: 19.77 (sum of all values ÷ 20)
Mean deviation of the distribution: 0.557 (sum of all mean deviations ÷ 20)

\[
\begin{align*}
|18.56 - 19.77| &= 1.21 \\
|18.7 - 19.77| &= 1.07 \\
&\vdots \\
|21.1 - 19.77| &= 1.33 \\
\end{align*}
\]
Sum of deviations = 11.14

\[11.14 ÷ 20 = 0.557\]

Swimmers that meet qualification 2.
PBT ≤ 20 – MD
PBT ≤ 20 – 0.557
PBT ≤ 19.443

They are: 18.56, 18.7, 18.9, 18.95, 19.2, 19.25, 19.26

Seven (7) swimmers will earn a spot on the National team.

Specific Strategies:

Remember that the lowest scores are the best scores. Therefore, the values in this distribution are given in order of best to worst, not from worst to best (which is what we usually see). This makes finding the percentile a little bit tricky.

Make sure that your answer takes into account the requirement to meet or exceed both of the evaluations that are used to select team members.

Additional Resources:

Visions Volume 1, pp. 81-82
2.2 Qualitative Interpretation of Correlation
Question:
Which of the following scatterplots shows the strongest linear correlation?

A)  

B)  

C)  

D)  

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank! Make a choice!

My Strategies:
### Answer and Solution:

Answers B) and D) can be ruled out since the points are quite far apart.

- **A)**
  ![Image A]
  The points are closest to forming a straight line but strongest in A)

- **B)**
  ![Image B]

- **C)**
  ![Image C]

- **D)**
  ![Image D]

The answer is A.

### Specific Strategies:

The scatterplot that shows a distribution of points closest to forming a straight line has the strongest correlation.

The strength of a correlation is not related to its direction (or sign).

### Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-95
Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:
- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity
Question:
Which of the following scatterplots best shows a zero correlation?

A) ![Scatterplot A]

B) ![Scatterplot B]

C) ![Scatterplot C]

D) ![Scatterplot D]

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!
Make a choice!

My Strategies:
Answer and Solution:

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>A slight positive line, so not 0</td>
</tr>
<tr>
<td>B)</td>
<td>A fairly significant clustering around a negative line</td>
</tr>
<tr>
<td>C)</td>
<td>A rather tight clustering around a positive line</td>
</tr>
<tr>
<td>D)</td>
<td>No tendency towards a line</td>
</tr>
</tbody>
</table>

The answer is D.

Suggested Strategies:

The scatterplot that shows a distribution of points with no clear direction and furthest from forming a straight line indicates a zero correlation.

Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-95
Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:
- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity
At a recent school event, students were asked to sit facing the stage. The following table shows the distribution of students according to their ages and the distance from the stage.

Which of the following best describes the linear correlation between the age of the students and the distance from each student to the stage?

A) The correlation is positive.

B) The correlation is negative.

C) The correlation is perfect.

D) The correlation is zero.
Answer and Solution:

<table>
<thead>
<tr>
<th>AGE (years)</th>
<th>DISTANCE (m)</th>
<th>[2, 4]</th>
<th>[4, 6]</th>
<th>[6, 8]</th>
<th>[8, 10]</th>
<th>[10, 12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[10, 11]</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>[11, 12]</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>[12, 13]</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>[13, 14]</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>[14, 15]</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

In a table, the closer the data is to the diagonal, the stronger the correlation.

All values are 3 so data is not clustered on the diagonal.

In this case, the data is evenly spread out, thus indicating no correlation.

A) The correlation is positive – false: data is not clustered around a diagonal from top left to bottom right.

B) The correlation is negative – false: data is not clustered around a diagonal from bottom left to top right.

C) The correlation is perfect – false: given an age, there is no way to predict the distance from the stage.

D) **The correlation is zero – true**: given an age, there is no way to predict the distance from the stage.

The answer is D.

Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-94
**Question:**

Consider the following table showing a two-variable distribution. Indicate the strength and direction of correlation.

<table>
<thead>
<tr>
<th>x</th>
<th>y [0,1]</th>
<th>[1,2]</th>
<th>[2,3]</th>
<th>[3,4]</th>
<th>[4,5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

<table>
<thead>
<tr>
<th>Strength</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak</td>
<td>Positive</td>
</tr>
<tr>
<td>Strong</td>
<td>Negative</td>
</tr>
</tbody>
</table>
**Answer and Solution:**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>[0,1]</th>
<th>[1,2]</th>
<th>[2,3]</th>
<th>[3,4]</th>
<th>[4,5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The answer is:

<table>
<thead>
<tr>
<th>Strength</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Weak</td>
<td>✓ Positive</td>
</tr>
<tr>
<td>✓ Strong</td>
<td>□ Negative</td>
</tr>
</tbody>
</table>

**Suggested Strategies:**

- In a table, the closer the data is to the diagonal, the stronger the correlation.
- If the diagonal slopes downward, then the correlation is positive – because as x increases, so does y.

**Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-94
Question:
Consider the following scatterplot.
A) Is the correlation weak or strong?
B) Is the direction positive or negative?

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:

<table>
<thead>
<tr>
<th><strong>Strength</strong></th>
<th><strong>Direction</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>□ Weak</td>
<td>□ Positive</td>
</tr>
<tr>
<td>□ Strong</td>
<td>□ Negative</td>
</tr>
</tbody>
</table>
**Answer and Solution:**

- The points appear to form a descending line (negative direction)
- The points are very scattered far apart (weak correlation)

**The answer is:**

<table>
<thead>
<tr>
<th>Strength</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>✅ Weak</td>
<td>☐ Positive</td>
</tr>
<tr>
<td>☐ Strong</td>
<td>✅ Negative</td>
</tr>
</tbody>
</table>

**Specific Strategies:**

- Look at how far or close the points are relative to each other
  - The closer the points are to forming a straight line, the stronger the correlation is
  - A positive slope means a positive correlation
  - A negative slope means a negative correlation

**Additional Resources:**

Visions Volume 1, Section 2.2, pp. 93-95
- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity
2.3 Quantitative Interpretation of Correlation
Question:

The table below shows the linear correlation coefficient between the two variables of four different statistical distributions.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Linear Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td>2</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>-0.27</td>
</tr>
<tr>
<td>4</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Which of the following presents these distributions, in order, from weakest to strongest linear correlation?

A) 1, 4, 2, 3
B) 3, 2, 4, 1
C) 1, 3, 2, 4
D) 4, 2, 3, 1

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

My Strategies:
**Answer and Solution:**

A) $-0.81, 0.74, 0.39, -0.27$

is from strongest to weakest

B) $-0.27, 0.39, 0.74, -0.81$

Correct

C) $-0.81, -0.27, 0.39, 0.74$

is from smallest number to greatest (keeps the sign)

D) $0.74, 0.39, -0.27, -0.81$

is from largest number to smallest number (keeps the sign)

The answer is B.

**Specific Strategies:**

Ignore the sign (+ or −)!

- Order the coefficients (without the sign) in increasing order.
- The coefficient closest to 0 is the weakest.
- The coefficient closest to 1 is the strongest.

**Additional Resources:**

Visions Volume 1, Section 2.3, p. 110

Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity
**Question:**

Which of the following correlation coefficient shows the weakest correlation?

<table>
<thead>
<tr>
<th>Choice</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>−0.75</td>
</tr>
<tr>
<td>B)</td>
<td>−0.45</td>
</tr>
<tr>
<td>C)</td>
<td>0.16</td>
</tr>
<tr>
<td>D)</td>
<td>0.83</td>
</tr>
</tbody>
</table>

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!
Make a choice!

**My Strategies:**
Answer and Solution:

A) $-0.75$ – don’t mistake the smallest number with the weakest correlation

B) $-0.45$ – is further from 0 than 0.16

C) 0.16 – correct

D) 0.83 – this is the strongest correlation shown

The answer is C.

Specific Strategies:

Ignore the + or – signs!

The coefficient closest to 0 is the weakest

Additional Resources:

Visions Volume 1, Section, 2.3 p. 110
Explore Learning Gizmos, http://www.explorelarning.com/ look up:
- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity
Question:

Which of the following correlation coefficients shows a perfect correlation?

A) \(-1.00\)
B) \(-0.10\)
C) 0.00
D) 0.99

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank! Make a choice!

My Strategies:
### Answer and Solution:

| A) | \(-1.00\) – correct (perfect correlation can be positive or negative) |
| B) | \(-0.10\) – in case you misplace the decimal |
| C) | \(0.00\) – in case you mix up “perfect” and “no correlation” |
| D) | \(0.99\) – really close, but not perfect. |

The answer is A.

### Additional Resources:

Visions Volume 1, Section 2.3, p. 110
Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:
- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

---

Specific Strategies:

- Ignore the + or – signs!
- A perfect correlation is equal to 1 or \(-1\)
Question:

Consider the following scatterplot.

What is the linear correlation coefficient?

The linear correlation coefficient is __________.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.
   Do not leave a blank!

My Strategies:
Answer and Solution:

\[ r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right) \]

\[ r = -\left( 1 - \frac{1.4 \text{ cm}}{5.3 \text{ cm}} \right) \]

\[ r = -(0.74) \]

Since the slope is negative, the correlation coefficient should also be negative.

The linear correlation coefficient is \(-0.74\).

*(Because of slight differences in measurement, your answer could be anywhere between \(-0.82\) and \(-0.66\).*

Additional Resources:

Visions Volume 1, Section 2.3, p. 110
Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity
Consider the following scatterplot.

Estimate the linear correlation coefficient.

The linear correlation coefficient is __________.
**Answer and Solution:**

\[ r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right) \]

\[ r = + \left( 1 - \frac{1.5 \text{ cm}}{4.5 \text{ cm}} \right) \]

\[ r = +(0.67) \]

Since the slope is positive, the correlation coefficient should also be positive.

The linear correlation coefficient is 0.67.

*(Because of slight differences in measurement, your answer could be anywhere between 0.59 and 0.75)*

**Specific Strategies:**

- Encase the points with a rectangle.
- Measure the dimensions of the rectangle.
- Estimate the linear correlation coefficient using the formula:

\[ r = \pm \left( 1 - \frac{\text{short side}}{\text{long side}} \right) \]

**Additional Resources:**

- Visions Volume 1, Section 2.3, p. 110
  - Correlation
  - Trends in Scatter Plots
  - Scatter Plots-Activity
**Question:**

The Granby Zoo feeds its elephants daily. The chart below shows the weight of several elephants and the weight of the food they are given every day.

<table>
<thead>
<tr>
<th>Weight of elephant (kg)</th>
<th>Weight of food (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1250</td>
<td>58</td>
</tr>
<tr>
<td>1300</td>
<td>63</td>
</tr>
<tr>
<td>1320</td>
<td>66</td>
</tr>
<tr>
<td>1382</td>
<td>69</td>
</tr>
<tr>
<td>1400</td>
<td>67</td>
</tr>
<tr>
<td>1460</td>
<td>63</td>
</tr>
<tr>
<td>1480</td>
<td>70</td>
</tr>
<tr>
<td>1492</td>
<td>76</td>
</tr>
</tbody>
</table>

How much food would an elephant weighing 1600 kg be given? Round your answer to the nearest tenth of a kilogram.

A 1600kg elephant would be given _________ kg of food.

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**
Perform a linear regression using the data in the table.

Mayer line method:
Let $x$ be the weight of an elephant, and $y$ be the weight of the food

Split data in half:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1250</th>
<th>1300</th>
<th>1320</th>
<th>1382</th>
<th>1400</th>
<th>1460</th>
<th>1480</th>
<th>1492</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>58</td>
<td>63</td>
<td>66</td>
<td>69</td>
<td>67</td>
<td>63</td>
<td>70</td>
<td>76</td>
</tr>
</tbody>
</table>

\[
\text{average } x_{\text{first half}} = \frac{1250 + 1300 + 1320 + 1382}{4} = 1313 \\
\text{average } x_{\text{second half}} = \frac{1400 + 1460 + 1480 + 1492}{4} = 1458 \\
\text{average } y_{\text{first half}} = \frac{58 + 63 + 66 + 69}{4} = 64 \\
\text{average } y_{\text{second half}} = \frac{67 + 63 + 70 + 76}{4} = 69 \\
\]

Point 1 (1313, 64)\hspace{2cm} Point 2 (1458, 69)

\[a = \frac{69 - 64}{1458 - 1313} = 0.0344\]
\[b = 64 - (0.0344)(1313) = 18.8328\]

Linear regression rule: \[y = 0.0344x + 18.8328\]

Find the value of $y$ when $x = 1600$
\[y = 0.0344(1600) + 18.8328 = 73.8728\]

A 1600 kg elephant would be given 73.9 kg of food. Your answer might vary.
2.4 Interpretation of Linear Correlation
**Question:**

Which of the following distributions suggests a linear correlation of the data?

A) ![Graph A]

B) ![Graph B]

C) ![Graph C]

D) ![Graph D]

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank! Make a choice!

**My Strategies:**
<table>
<thead>
<tr>
<th>Answer and Solution:</th>
<th>Suggested Strategies:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A)</strong></td>
<td>The graph that shows data points that closely form a straight line yields the best interpretations for linear correlation</td>
</tr>
<tr>
<td><img src="image" alt="Graph A" /></td>
<td><strong>This data is clustered along a line</strong></td>
</tr>
<tr>
<td><strong>B)</strong></td>
<td><strong>This data is clustered along a curve (not linear)</strong></td>
</tr>
<tr>
<td><img src="image" alt="Graph B" /></td>
<td><strong>This data has a large gap and therefore a straight linear correlation can’t be assumed</strong></td>
</tr>
<tr>
<td><strong>C)</strong></td>
<td><strong>This data is clustered in one area rather than along a line</strong></td>
</tr>
<tr>
<td><img src="image" alt="Graph C" /></td>
<td><strong>This data is clustered along a curve (not linear)</strong></td>
</tr>
</tbody>
</table>

The answer is **A**.

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124
Question:

Four different relationships between two variables are shown in the graphs below.

Which relationship would allow you to make the most accurate prediction?

A)

B)

C)

D)

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

B) – the data points in this graph are closest to forming a line.

**Suggested Strategies:**

The graph showing the strongest correlation, whether positive or negative, would demonstrate the strongest statistical link between two variables. A strong statistical link leads to better prediction.

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124
[http://www.purplemath.com/modules/scattreg2.htm](http://www.purplemath.com/modules/scattreg2.htm)
Question:
The linear correlation coefficient between two variables is -0.93.

Which of the following best describes the correlation?

A) The correlation between the two variables is strong and positive.
B) The correlation between the two variables is strong and negative.
C) The correlation between the two variables is weak and positive.
D) The correlation between the two variables is weak and negative.

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

My Strategies:
**Answer and Solution:**

A) False: The correlation between the two variables is strong and **not** positive.

B) The correlation between the two variables is strong and negative.

C) False: The correlation between the two variables is **not** weak and positive.

D) False: The correlation between the two variables is **not** weak and negative.

**Suggested Strategies:**

The options are all wordy but you will see that they are almost exactly the same; look for the differences – you might want to use a highlighter...

The correlation coefficient of \(-0.93\) is close to \(-1\), this results in a correlation described as negative and strong.

**The answer is B.**

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124


[http://www.dummies.com/how-to/content/how-to-interpret-a-correlation-coefficient-r.html](http://www.dummies.com/how-to/content/how-to-interpret-a-correlation-coefficient-r.html)
**Question:**

The linear correlation between two variables is positive and weak. Which of the following could represent the correlation coefficient?

A) 0.32  
B) 0.87  
C) −0.26  
D) −0.91

**General Strategies:**

1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Look carefully at each choice shown (A, then B, then C and then D).  
7. Eliminate options you know to be incorrect.  
8. Solve/check each possible choice.  
9. Select the choice that makes the most sense.  

Do not leave a blank! Make a choice!

**My Strategies:**
### Answer and Solution:

<table>
<thead>
<tr>
<th>Option</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>0.32</td>
<td>close to 0 and positive</td>
</tr>
<tr>
<td>B)</td>
<td>0.87</td>
<td>positive but not close to 0</td>
</tr>
<tr>
<td>C)</td>
<td>−0.26</td>
<td>closest to 0 but negative</td>
</tr>
<tr>
<td>D)</td>
<td>−0.91</td>
<td>negative and not close to 0</td>
</tr>
</tbody>
</table>

The answer is A.

### Suggested Strategies:

A weak and positive correlation is best represented by a correlation coefficient value that is positive and much closer to 0 than to 1.

### Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Consider the scatterplot below. Which of the following could best describe the linear correlation between the two variables?

A) Strong and positive
B) Strong and negative
C) Weak and positive
D) Weak and negative
**Answer and Solution:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>Strong and positive</td>
</tr>
<tr>
<td>B)</td>
<td>Strong and negative</td>
</tr>
<tr>
<td>C)</td>
<td>Weak and positive</td>
</tr>
<tr>
<td>D)</td>
<td>Weak and negative</td>
</tr>
</tbody>
</table>

The answer is D.

**Specific Strategies:**

- **This scatterplot shows data points that trend downward suggesting a negative correlation.**

- **This scatterplot also shows data points that are spread apart rather than close together (along a line) suggesting a weak correlation.**

**Additional Resources:**

- Visions Volume 1, Section 2.4, pp. 123-124
Question:

Consider the scatterplot below. Which of the following could best describe the linear correlation coefficient between the two variables?

A) 0.29  
B) 0.83  
C) −0.45  
D) −0.79

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

My Strategies:
**Answer and Solution:**

A) 0.29 – positive by not high enough so suggest a strong correlation

B) 0.83 – positive and high enough to suggest a strong correlation

C) −0.45 – a negative correlation, and not very strong

D) −0.79 – a negative correlation, even though it is fairly strong.

**Suggested Strategies:**

This scatterplot shows data points that trend upward therefore you are looking for a positive correlation coefficient.

This scatterplot also shows data points that are close together (along a line) rather than spread out so you are looking for a correlation coefficient that suggests a strong correlation.

**The answer is B.**

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124


Question:

A class of secondary 4 students measured their foot lengths and their heights. The results are represented in the table below.

<table>
<thead>
<tr>
<th>Foot length (cm)</th>
<th>Height (cm)</th>
<th>Foot length (cm)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>154</td>
<td>25.5</td>
<td>170</td>
</tr>
<tr>
<td>22</td>
<td>151</td>
<td>25.5</td>
<td>173</td>
</tr>
<tr>
<td>23</td>
<td>155</td>
<td>26</td>
<td>167</td>
</tr>
<tr>
<td>23.5</td>
<td>165</td>
<td>27</td>
<td>174</td>
</tr>
<tr>
<td>24</td>
<td>160</td>
<td>27.5</td>
<td>175</td>
</tr>
<tr>
<td>24</td>
<td>158</td>
<td>28</td>
<td>176</td>
</tr>
<tr>
<td>24.5</td>
<td>165</td>
<td>28</td>
<td>183</td>
</tr>
<tr>
<td>25</td>
<td>161</td>
<td>28.5</td>
<td>185</td>
</tr>
<tr>
<td>25</td>
<td>163</td>
<td>29</td>
<td>190</td>
</tr>
<tr>
<td>25.5</td>
<td>164</td>
<td>29.5</td>
<td>186</td>
</tr>
</tbody>
</table>

Marco was absent the day data was collected. School records show that Marco is 181 cm tall.

What is the predicted length of Marco’s foot?

Marco’s predicted foot length is ___________.

General Strategies:

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

My Strategies:
**Answer and Solution:**

With the Mayer line method:

- $x$ represents the foot length in centimetres
- $y$ represents the height in centimetres

Since the foot lengths are already in ascending order this step has been done for us.

\[ x_1 = \frac{22+22+23+23.5+24+24+24.5+25+25+25.5}{10} = 23.85 \]

\[ x_2 = \frac{25.5+25.5+26+27+27.5+28+28.5+29+29.5}{10} = 27.45 \]

\[ y_1 = \frac{154+151+155+160+158+165+161+163+164+164}{10} = 159.6 \]

\[ y_2 = \frac{170+173+167+174+175+176+183+185+190+186}{10} = 177.9 \]

\[ a = \frac{177.9 - 159.6}{27.45 - 23.85} = \frac{18.3}{3.6} = 5.08 \]

\[ y = 5.08x + b \]

using either point (23.85, 159.6)

\[ 159.6 = 5.08(23.85) + b \]

or (27.45, 177.9)

\[ 177.9 = 5.08(27.45) + b \]

\[ b = 38.44 \]

\[ y = 5.08x + 38.44 \]

Marco: \[ 181 = 5.08x + 38.44 \]

\[ 181 - 38.44 = 5.08x \]

\[ 142.56 = 5.08x \]

\[ x = \frac{142.56}{5.08} = 28 \text{ cm} \]

**Answer:** Marco’s predicted foot length is **28 cm**. (note that if you use a different method you will get a slightly different answer)

---

**Suggested Strategies:**

What you are looking for is a linear equation relating height and foot length. Once you’ve found one, you will use it to find foot length, knowing height.

There are a number of methods possible.

- Complete a scatter plot, drawing in the line of best fit and finding the equation of that line
- Use the median-median method
- Use the Mayer line method
- Enter the data into a graphing calculator to get the regression line

---

**Additional Resources:**

Visions Volume 1, Section 2.4, pp. 123-124
3.1 Congruent Triangles
Question:
Which of the following pairs of triangles is not necessarily congruent?

A)  

B)  

C)  

D)  

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!
Make a choice!

My Strategies:
**Question 50**

<table>
<thead>
<tr>
<th>Answer and Solution:</th>
<th>Suggested Strategies:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A)</strong> This is the correct answer. These triangles are <em>not necessarily</em> congruent since they only have two congruent sides and no congruent angles indicated.</td>
<td>Check all possible answers, and beside each one write the proof that confirms congruency. You should be left with only one that has no proof (meaning those triangles are <em>not</em> congruent).</td>
</tr>
<tr>
<td><strong>B)</strong> These are congruent by ASA. Also, since two angles are congruent, then the third pair is too.</td>
<td></td>
</tr>
<tr>
<td><strong>C)</strong> These are congruent by SSS.</td>
<td></td>
</tr>
<tr>
<td><strong>D)</strong> These are congruent by ASA.</td>
<td></td>
</tr>
</tbody>
</table>

The answer is A.

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161


Question:
Consider the following diagram.

What theorem can be used to show that ΔABD is necessarily congruent to ΔACD?

A) SSS
B) SAS
C) ASA
D) AA

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.
Do not leave a blank!
Make a choice!

My Strategies:
**Answer and Solution:**

The diagram identifies pairs of congruent angles and even though the congruent sides are not identified, the triangles share a side, which makes it congruent. Since the shared side is between pairs of congruent angles, the proof ASA is valid to prove congruency.

The answer is C.

**Suggested Strategies:**

When proving congruency (≅), first consider the three possible proofs (SSS, SAS, ASA). Starting with this will likely help you to eliminate one or two of the proofs as not having enough information pretty quickly.

Although this proof is perhaps not obvious since the congruent sides aren’t identified, don’t overlook the fact that the triangles share a side (AD).

**Additional Resources:**

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161


Question

Which of the following pairs of triangles is necessarily congruent?

A) 

B) 

C) 

D) 

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!
Make a choice!

My Strategies:
Answer and Solution:

Although the top angle is not identified as being congruent in the two triangles in answer b, they must be since the other two pairs of corresponding angles are the same. Since the unidentified angles are now known to be congruent, two theories can be used to prove congruency – ASA and SAS.

A) There is not enough information, having two angles the same makes the triangle similar but not necessarily congruent

B) This may look like ASA but the congruent sides are not between the congruent angles so you can’t conclude the triangles are congruent by that theory.

C) This is the correct answer.

D) This pair doesn’t have corresponding sides that are congruent so you can’t conclude they are congruent by ASA.

The answer is C.

Suggested Strategies:

Don’t be discouraged if the correct answer doesn’t jump out at you right away! Even though this is a multiple choice question, and you might expect to see the answer right away, there is often work or extra thinking needed to uncover the correct answer. Don’t give up until you’ve tried all possibilities, in this case it involved a little extra thought.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Khan Academy video:
http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS)
Question:

Point P is the midpoint of line segment MN. Segment ML is parallel to segment ON. Triangles ΔLMP and ΔONP are congruent.

Show that triangles ΔLMP and ΔONP are congruent. Remember to justify each of your statements.

\[ \triangle LMP \cong \triangle ONP \] by ____________

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

\( MP \cong NP \) since P is the midpoint of MN, then MP and NP are congruent.

\(<LPM \cong <OPN \) \(<LPM \) and \(<OPN \) are vertically opposite and therefore congruent.

\(<PML \cong <PNO \) \(<PML \) and \(<PNO \) are alternate interior angles of a transversal through parallel lines, which means those angles are congruent.

With this information, we can say the triangles are necessarily congruent using the ASA theorem.

\[ \triangle LMP \cong \triangle ONP \text{ by ASA} \]

Suggested Strategies:

Make sure you fill in all the information that you know on your diagrams. This is important on all questions of a test, but especially on ones where there is obviously information that has been left out. Don’t forget all of the angle relationships when filling in information – and seeing a transversal through parallel lines should remind you of those angle relationships.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Khan Academy video: http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS)
**Question:**

Consider the following diagram.

Show that \(\triangle ABC\) is necessarily congruent to \(\triangle EDC\).

\[
\begin{align*}
\triangle ABC & \cong \triangle EDC \\
\text{by } & \quad \text{(Reason)}
\end{align*}
\]

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**
Answer and Solution:

This diagram shows only two pairs of sides are congruent so you can eliminate ASA.

That leaves SSS and SAS. But you know that vertically opposite angles are necessarily congruent even if they aren’t identified.

\[
\overline{AC} \cong \overline{EC} \\
\angle ACB \cong \angle ECD \\
\overline{BC} \cong \overline{DC}
\]

\[\triangle ABC \cong \triangle EDC\] by SAS

Suggested Strategies:

When proving congruency consider the three possible proofs (SSS, SAS, ASA). Starting with this will likely help you to eliminate one or two of the proofs as not having enough information pretty quickly.

In other cases, don’t forget to go through the possible angle relationships for intersecting and transverse lines across parallel lines.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Khan Academy video: http://www.khanacademy.org/search?page_search_query=congruent+triangles
Congruent triangles (all conditions: SSS, ASA and SAS
Question:
You’ve been hired to paint a team logo on a field for a sporting event. The logo consists of two congruent triangles.

Here is the information you have:

- Point $P$ is the midpoint of $MN$
- $M$ is located at $(-60, 48)$
- $N$ is located at $(28, 4)$
- $\angle LPM$ measures $95^\circ$
- $\angle LMP$ measures $15^\circ$

**measurements are in metres**

You charge $5 per $m^2$ for doing the painting (round your measurements to the nearest $m^2$).

What will you charge in total for painting the two triangles that make up the logo?

You will charge ____________ for painting the two triangles that make up the logo.
Answer and Solution:

Step 1) Calculate the coordinates of P using the midpoint formula. They are (-16, 26).

\[
\begin{align*}
(x_m, y_m) &= \left(\frac{-60+28}{2}, \frac{48+4}{2}\right) \\
(x_m, y_m) &= \left(\frac{-32}{2}, \frac{52}{2}\right) \\
(x_m, y_m) &= (-16, 26)
\end{align*}
\]

OR Calculate the distance between the two points and divide that distance by 2.

Step 2) Calculate the distance from M to P. It is 49.1935 m.

\[
d = \sqrt{(-16 - -60)^2 + (26 - 48)^2} \\
d = 49.1935 \\
d = 24.20 \\
d = 49.1935 \\
\]

Step 3) The missing angle (<MLP) is 70°. Using the Sine Law, calculate the missing measurements for ΔLMP. We know side MP is 49.1935 m, we can calculate side LM. It is 52.15 m. We can then calculate side LP. It is 13.55 m.

\[
x = \frac{49.1935 \sin 70}{\sin 15} \\
x = 13.55 \text{ metres (side LP)} \\
y = \frac{49.1935 \sin 95}{\sin 70} \\
y = 52.15 \text{ m (side LM)}
\]

Step 4) Calculate the area of ΔLMP using either the Trig Formula or Hero’s Formula. It is 332 m² (rounded to the nearest square metre).

Example of Trig Formula:

\[
\text{Area} = \frac{(13.55)(52.15) \sin 70}{2} = 332 \text{ m}^2
\]

OR Hero’s Formula:

Half the perimeter: \[\frac{49.19+13.55+52.15}{2} = 57.45\]

\[
\text{Area} = \sqrt{57.45(57.45 - 49.19)(57.45 - 13.55)(57.45 - 52.15)} \\
\text{Area} = 332 \text{ metres}^2
\]

Step 5) Multiply the area of ΔLMP by 2, then multiply that by $5.

The total cost of painting is $3320.

\[
(332m^2 \times 2 \times $5/m^2 = $3320)
\]

You will charge $3320 for painting the two triangles that make up the logo.

Suggested Strategies:

You need to find the area of the triangles and multiply that by $5.

What do you need to find the area of a triangle?

- The length of a base and altitude or
- The length of two sides and the angle between them (Trig area formula) or
- The length of all three sides (Hero’s Formula)

Choose the method you think will work for you and find the measures you need.

You only need to do this once since the triangles are congruent.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Khan Academy video: http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS
http://mathbits.com/MathBits/TISection/Trig/AreaTrigTri.htm
3.2 Similar Triangles
Question:

The diagram below illustrates two triangles in which the measurements of some sides are given and $\overline{AB} \parallel \overline{DE}$.

What is the length of segment $BE$?

E) 2.5 cm
F) 3.75 cm
G) 14.5 cm
H) 15.75 cm

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank!
Make a choice!

My Strategies:
Answer and Solution:

$\triangle ABC \sim \triangle DEC$ because of the AA theorem – angle C is the same in both triangles, and because they are vertically opposite, $\angle E \cong \angle B$ (and $\angle D \cong \angle A$) because they are alternate interior angles.

In order to determine the total length of $BE$, we need the length of $CE$, so label the measure of $CE$ as “$x$”.

Since the triangles are similar, their sides must be proportional. Set up a proportion using corresponding sides:

$$\frac{mCE}{mBC} = \frac{mED}{mAB} \rightarrow \frac{x}{12} = \frac{5}{16}$$

Cross multiply to determine the value of $x$

$$x = 12 \times 5 \div 16 = 3.75 \text{ cm}$$

To determine the length of side $BE$, add $mBC + mCE$

$$12 + 3.75 = 15.75 \text{ cm}$$

A) 2.5 cm is the measure of segment CD.

B) 3.75 cm is the measure of segment EC. The measure of segment BC must be added to this.

C) The result of adding the measures of segments CD and BC instead of EC and BC.

D) 15.75 cm is correct.

The answer is D.

Specific Strategies:

Some distractors might stand out: choices A) and B) are both fairly small, whereas C) and D) are both fairly large. An educated guess would eliminate A) and B), but we should remember that the drawings are never to scale.

- Label the figure with the given measurements
- Recognize that the triangles are similar because of AA

Alternate strategy:

Determine the scale factor $k$ by dividing the lengths of corresponding sides: $k = \frac{3.2}{3}$

Divide side BC by 3.2 to get the length of side $CE = 3.75 \text{ cm}$

- After completing the calculations, re-read the question and re-read the choices

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems) Similarity example problems
Question:

Triangle XYZ is shown below.

Which of the triangles below is necessarily similar to triangle XYZ?

A)  
\[ \begin{align*} 
&12\text{cm} \\
&27^\circ \\
&13\text{cm} \\
\end{align*} \]

B)  
\[ \begin{align*} 
&6\text{cm} \\
&7\text{cm} \\
&3\text{cm} \\
\end{align*} \]

C)  
\[ \begin{align*} 
&48^\circ \\
&27^\circ \\
&27\text{cm} \\
\end{align*} \]

D)  
\[ \begin{align*} 
&8\text{cm} \\
&48^\circ \\
&6\text{cm} \\
\end{align*} \]

General Strategies:
1. Read the question
2. Highlight key words
3. Identify the math topic
4. Re-read the question
5. Refer to your memory aid, as needed
6. Look carefully at each choice shown (A, then B, then C and then D)
7. Eliminate options you know to be incorrect
8. Solve/check each possible choice
9. Select the choice that makes the most sense

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

$m \angle y = 180^\circ - 105^\circ - 27^\circ = 48^\circ$ since the angles in every triangle add up to $180^\circ$

A) Incorrect: This triangle only has one angle in common with triangle XYZ. Two side measures are given, but it is only possible to compare with one side of XYZ and we need at least two sides to prove SSS or SAS. (If you used the Sine Law to find the missing side of the original triangle, you will also see that the sides are not proportional.)

B) Incorrect: This triangle gives us three side measurements, but like option A, we would need to be able to compare at least two sides to prove SSS or SAS.

C) Correct: This triangle is similar to XYZ. It has two angles in common because the third angle was calculated above to be $48^\circ$. So by AA, the triangles are similar.

D) Incorrect: This triangle is not necessarily similar to XYZ because the information the triangle is not unique – you can make many triangles with those three features fixed.

The answer is C.

Specific Strategies:

- Keyword: Similar
- Recall theorems on similar triangles
- Determine the measure of the third angle in triangle XYZ
- Triangles are similar if they satisfy one of three theorems – AA, SSS or SAS

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems) Similarity example problems
Question:

In the diagram below, $\overline{BD}$ and $\overline{AE}$ intersect at $C$

Other measurements are given:

$m\overline{AC} = 30m$
$m\overline{BC} = 25m$
$m\overline{CD} = 15m$
$m\overline{CE} = 18m$

Which of the following statements could be used to prove that triangle $ABC$ is similar to triangle $EDC$?

A) Two triangles are similar if they have two congruent corresponding angles. (AA)

B) Two triangles are similar if they have corresponding sides that are proportional. (SSS)

C) Two triangles are similar if they have corresponding proportional contained between two congruent angles. (ASA)

D) Two triangles are similar if they have a congruent angle contained between the corresponding sides that are proportional. (SAS)
**Answer and Solution:**

\( \angle C \cong \angle C \) because they are vertically opposite angles in the original diagram.

Because two corresponding pairs of sides are given, as well as the angle contained, we should consider that the triangles are similar by SAS.

Check if the sides are proportional:

\[
\frac{30}{18} = \frac{25}{15}
\]

A) We are only given one pair of corresponding angles and we don’t know for sure that side \( \overline{AB} \) is parallel to side \( \overline{DE} \), so it’s not AA.

B) Only 3 side lengths are given so we cannot consider SSS.

C) ASA is not a theorem for similarity.

D) Because the proportions of two pairs of sides are the same and the angle contained by those sides is congruent, the triangles are similar by SAS.

The answer is D.

**Specific Strategies:**

- Keyword: Similar
- Recall theorems on similar triangles
- Re-draw the triangles in the same orientation – be careful with the rotation
- Take note of other measurements which are not given in the problem
- Consider each of the similarity theorems

**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems) Similarity example problems
Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similar-triangle-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similar-triangle-example-problems) Similar triangle example problems
Question:

In the figure below, triangles ABC and ADE are similar.

\( \overline{DE} \) is parallel to \( \overline{BC} \),

\( \overline{mDE} = 20 \text{ m} \),
\( \overline{mBC} = 35 \text{ m} \)

and
\( \overline{mAE} = 24 \text{ m} \).

What is the length of segment EC?

The length of segment EC is \( \frac{20}{35} \times 24 \) m.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

Because the unknown length is part of side $\overline{AC}$, it will be simpler to solve for the length of side $\overline{AC}$ first and then subtract 24m.

Set up a proportion using corresponding sides:

\[
\frac{mDE}{mBC} = \frac{mAЕ}{mAC}
\]

\[
\frac{20}{35} = \frac{24}{mAC}
\]

\[
mAC = 42
\]

If side AC measures 42 m, then the length of EC is:

\[
42 - 24 = 18 \text{ m}
\]

Answer: The length of segment EC is 18 m.

Specific Strategies:

- Keyword: Similar
- Label measurements directly on the figure
- Highlight the length that the question is asking for
- Re-draw the triangles separately with the same orientation and include any measurements given
- Highlight corresponding sides with different colors
- Label the unknown measurement as “x”
- Reminder: re-do your calculations to double check, you will not get partial credit if your final answer is wrong!

Additional Resources:

Visions Volume 1, section 3.2, p. 171 (Minimum Conditions for Similar Triangles)

Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-example-problems) Similarity example problems
**Question:**

The following information is given about right triangles ABC and DEF:

What is the measure of $\angle D$?

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

The measure of $\angle D$ is ______.
Answer and Solution:

\[ m\angle D = 180° - 90° - 25° = 65° \]

Answer: The measure of \( \angle D \) is 65°.

Specific Strategies:

- Label the given measurements on the figure
- Draw the figures side by side in the same orientation
- Because the triangles have two angles in common, they must be similar by the AA theorem
- If two triangles have two angles in common, their third angle must also be common.

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 – Minimum Conditions for Similar Triangles
Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similar-triangle-basics](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similar-triangle-basics) Similar triangles basics
Question:

The following measures are given for the figure below:

\[ \overline{AD} = 12\text{cm} \]
\[ \overline{DB} = 4\text{cm} \]
\[ \overline{AE} = 8\text{cm} \]
\[ \overline{EC} = 16\text{cm} \]

Is triangle \( \triangle ABC \) similar to \( \triangle AED \)?

Note: The figure is not necessarily drawn to scale.

General Strategies:
1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

My Strategies:

☐ Yes, triangle \( \triangle ABC \) is similar to \( \triangle AED \)

☐ No, triangle \( \triangle ABC \) is not similar to \( \triangle AED \)
**Answer and Solution:**

![Diagram of triangles](image)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Justification</th>
</tr>
</thead>
</table>
| The ratio of one pair of corresponding sides = 2                         | \[
\frac{m\overline{AC}}{m\overline{AD}} = \frac{24\text{cm}}{12\text{cm}} = 2\]
| \(\angle A \cong \angle A\)                                              | Corresponding angles are congruent                                             |
| The ratio of a second pair of corresponding sides = 2                    | \[
\frac{m\overline{AB}}{m\overline{AE}} = \frac{16\text{cm}}{8\text{cm}} = 2\]
| \(\Delta ABC \sim \Delta AED\)                                          | SAS (Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar) |

- Yes, triangle \(\Delta ABC\) is similar to \(\Delta AE\)
- No, triangle \(\Delta ABC\) is not similar to \(\Delta AED\)

**Specific Strategies:**
- Label the figure with the given measurements
- Highlight the task
- Re-draw the triangles separately, with the same orientation
- Label the figure with the given measurements
- Label any other known measurements — we know that angle A measures the same in both triangles
- Highlight corresponding sides of the triangles
- Determine which geometric statement will allow you to prove the triangles are similar — SAS

**Additional Resources:**

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video: [http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-postulates](http://www.khanacademy.org/math/geometry/similarity/triangle_similarity/v/similarity-postulates) Similarity postulates
3.3 Metric Relations (Right Triangles)
Question:

The following information about triangle ABC below is known:

- \( \angle ABC = 90^\circ \)
- \( BD \) is an altitude
- \( AB = 60 \text{ m} \)
- \( BC = 80 \text{ m} \)

What is the measure of altitude \( BD \)?

A) 36 m  
B) 48 m  
C) 64 m  
D) 69 m  

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.  
Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

Step 1: Pythagorean Theorem

\[(m_{AC})^2 = 60^2 + 80^2\]
\[m_{AC} = \sqrt{3600 + 6400}\]
\[m_{AC} = 100 \text{ m}\]

Step 2: Apply Metric Relation

\[a \cdot b = c \cdot h\]
\[(m_{CB})(m_{AB}) = (m_{AC})(m_{BD})\]
\[60(80) = 100(m_{BD})\]
\[m_{BD} = \frac{4800}{100}\]
\[m_{BD} = 48 \text{ m}\]

A) 36 m - this is \(m_{AD}\)

B) 48 m - correct

C) 64 m - this is \(m_{DC}\)

D) 69 m - this is the result from using an incorrect formula

The answer is B.

**Suggested Strategies:**

1) Put the measures onto the diagram.
2) If you are using formulas, make sure you label the triangle according to your formulas.
3) Identify the metric relation(s) that enable(s) you to solve for the unknown.
4) Consider that it might be more than one step.

*Note: In this case you must apply Pythagorean theorem first before applying a metric relation formula.*

**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181
Question:

Triangle ABC has the following properties:

- $m \angle ACB = 90^\circ$
- $\overline{DC}$ is an altitude
- $m \overline{DB} = 45 \text{ m}$
- $m \overline{AD} = 12 \text{ m}$

What is the area of $\triangle ABC$?

A) $139 \text{ m}^2$
B) $523 \text{ m}^2$
C) $662 \text{ m}^2$
D) $1325 \text{ m}^2$

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

Step 1: Metric Relation

\[ h^2 = m \cdot n \]
\[ (mCD)^2 = (mAD) \cdot (mDB) \]
\[ mCD = \sqrt{12 \cdot 45} \]
\[ mCD = \sqrt{540} \]
\[ mCD = 23.2379 \text{ meters} \]

Step 2: Area of Triangle ABC

\[ \frac{mCD(mAD + mDB)}{2} \]
\[ \frac{23.2379(12 + 45)}{2} \]
\[ 662 \text{ m}^2 \]

The answer is C.

Suggested Strategies:

1) Orient the triangle in a way that is easiest for you.

2) Put the measures on the diagram and re-label if necessary.

3) Select the appropriate metric relation formula. In this case we need ‘h’ and we are given ‘m’ and ‘n’.

4) Remember your basic area formulas. In this case we need area of triangle.

\[ \text{Area} = \frac{\text{base} \times \text{height}}{2} \]

Also be sure you get the area of the requested triangle. In this case the questions ask for triangle ABC – the largest of the three triangles.

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
[http://www.youtube.com/watch?v=fdEBhf9SOYA](http://www.youtube.com/watch?v=fdEBhf9SOYA)
Question:

A construction crane pictured below, has the following measurements:

- $m \angle TWP = 90^\circ$
- $VW$ is an altitude
- $m \overline{VW} = 50$ metres
- $m \overline{TV} = 70$ metres

What is the measure of angle WPV to the nearest tenth of a degree?

The measure of angle WPV is __________.
**Answer and Solution:**

Use Metric Relations to find the missing side.

\[ h^2 = m \cdot n \]
\[ 50^2 = 70(mP\overline{V}) \]
\[ \frac{50^2}{70} = mP\overline{V} \]
\[ \frac{250}{7} = mP\overline{V} \]
\[ mP\overline{V} = 35.714 \]

Use a Trigonometric Ratio to find the missing angle.

\[ \tan^{-1} \left( \frac{50}{35.714} \right) = 54.46^\circ \]

The measure of angle WPV is \( \approx 54.5^\circ \).

**Suggested Strategies:**

1) Redraw the diagram with only the triangles and measures (so the crane doesn’t confuse you.) Now locate angle WPV.

2) If we find the measure of PV, we can use a trig ratio (tangent) to find the missing angle.

3) To find the missing measure we’ll use a metric relation that uses the measures we have.

**Make sure your calculator is set to degree mode.**

**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181
Question:
Triangle ABC has the following properties:

- $m \angle BCA = 90^\circ$
- $\overline{CD}$ is an altitude
- $m \overline{AB} = 20 \text{ m}$
- $m \overline{BC} = 10 \text{ m}$

What is the measure of $\overline{AD}$?

The measure of $\overline{DA}$ is ________.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
**Answer and Solution:**

Step 1: Solve for $mDB$

\[ a^2 = n \cdot c \]
\[ (mBC)^2 = (mBA)(mDB) \]
\[ 10^2 = 20(mDB) \]
\[ 100 = (mDB) \]
\[ 20 = 5 \]

Step 2: Solve for $(mAD)$

\[ (mAB) = mAD + mDB \]
\[ 20 = mAD + 5 \]
\[ mAD = 15 \]

The measure of $AD$ is 15 m.

**Suggested Strategies:**

1) If necessary, orient the triangle in a way that is easiest for you.

2) If necessary, re-label the vertices of triangle according to your memory aid.

3) Add the numerical information to the diagram.

4) Identify the metric relation formula that enables you to solve for the unknown or set up a proportion knowing the three triangles are similar:

\[ \frac{\text{hyp}_{\text{small}}}{\text{short side}_{\text{small}}} = \frac{\text{hyp}_{\text{large}}}{\text{short side}_{\text{large}}} \]

\[ \frac{10}{BD} = \frac{20}{10} \]

**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181
**Question:**

A group of Brazilian soccer players are practicing their passes before a game. Their coach illustrates on a Cartesian plane a possible game scenario by showing the players as vertices of three similar right angle triangles.

The coach places the player Zico (Z) on the sideline (y-axis) to perform a throw-in to the player Kaka (K), who would pass the ball to Pele (P) located at the coordinates (15, 60) followed by a pass to Falcao (F) located at (60, 90). Units are in metres.

What is the total combined distance of all three passes?

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

**My Strategies:**

The total combined distance is ____________ m.
Answer and Solution:

Step 1: Solve $m_{PF}$
Distance formula \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)
P(15, 60) and F(60, 90) \(\sqrt{(60 - 15)^2 + (90 - 60)^2} = m_{PF}\)
\[m_{PF} = \sqrt{45^2 + 30^2} = 54.08\]

Step 2: Solve $m_{ZP}$
Set-up equation of line ZF \(y = ax + b\)
Given P(15, 60) and F(60, 90)
\[
a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 60}{60 - 15} = \frac{30}{45} = \frac{2}{3}
\]
\[y = \frac{2}{3}x + b\quad \text{Now we have to solve for 'b'.}
\]
Solve for 'b' by substituting the coordinates of a point on the line into the equation.
In this case we have a choice between point P and F. It does not matter which one you choose.
Using the coordinates of point P we have,
\[
60 = \frac{2}{3}(15) + b
\]
\[60 = 10 + b
\]
\[b = 50
\]

Step 3: Solve $m_{ZP}$
Distance formula \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)
Z(0, 50) and P(15, 60)
\[
\sqrt{(15 - 0)^2 + (60 - 50)^2} = m_{PZ}
\]
\[m_{PZ} = \sqrt{15^2 + 10^2} = 18.03
\]

Step 4: Metric Relations to solve $m_{PK}$
\[h^2 = m \cdot n
\]
\[(m_{PK})^2 = (m_{ZP})(m_{PF})
\]
\[m_{PK} = \sqrt{18.03(54.08)}
\]
\[m_{PK} = 31.23
\]

Step 5: Solving for $m_{ZK}$
\[
m_{ZK} = \sqrt{(m_{ZP})^2 + (m_{PK})^2}
\]
\[m_{ZK} = \sqrt{18.03^2 + 31.23^2}
\]
\[m_{ZK} = 36.06
\]

Step 6: Sum up the lengths of three line segments
\[m_{ZK} + m_{PK} + m_{PF} = 121.37 \text{ meters}
\]

The total combined distance is **121.37 m**.

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
Question:

A group of engineers is planning the construction of the new Champlain Bridge in Montreal. Below is a diagram of a section of the bridge.

The bridge’s towers (BD and EC) are each 100 metres in height and one of the support cables (AB) measures 110 metres.

Also, $m \angle ABC$ and $m \angle DEF$ are both 90° and the towers are perpendicular to the base of the bridge.

Show that the length of the cable represented by segment DE measures 240 m.

General Strategies:

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

My Strategies:
Answer and Solution:

*Here is an example of an appropriate method*

![Diagram of triangles BCD and ECD with lengths labeled]

**Step 1:** We can prove that \( \triangle BCD \) and \( \triangle ECD \) are congruent by theorem SAS:

**Justification:**
- \( m\overline{CD} \) is a shared length to both triangles
- \( m\overline{BD} = m\overline{CE} \) Given information
- \( m \angle ABC \) and \( m \angle DEF \) are both 90°

\( m\overline{BC} = m\overline{DE} \) since they are corresponding sides of congruent triangles.

**Step 2:** Apply Pythagorean Theorem to find \( m\overline{AD} \)

\[
m\overline{AD} = \sqrt{110^2 - 100^2} = 45.8258
\]

**Step 3:** Apply Metric Relation to find \( m\overline{DC} \)

\[
h^2 = m \cdot n
\]
\[
(m\overline{BD})^2 = (m\overline{AD})(m\overline{CD})
\]
\[
100^2 = 45.8258(m\overline{CD})
\]
\[
100^2 = 45.8258 \Rightarrow (m\overline{CD}) = 218.2177
\]

**Step 4:** Apply Pythagorean Theorem to find \( m\overline{BC} \)

\[
(m\overline{BC}) = \sqrt{218.2177^2 + 100^2} = 240 \text{ metres}
\]

The length of the cable represented by \( \overline{DE} \) is indeed 240 m.

**Suggested Strategies**

1) Put the numerical information into the diagram.
2) Since triangle DEF has no information about it and triangle DEC only has one measure, convince yourself that the triangles with more measures can be used.
3) See that \( \overline{BC} \) is the same length as \( \overline{ED} \) so if you find \( m\overline{BC} \) you’ll know \( m\overline{DE} \).
4) Use Pythagoras to find \( m\overline{AD} \).
5) Use a metric relation to find \( m\overline{DC} \).
6) Use Pythagoras again to find \( m\overline{BC} \).
7) \( \overline{BC} \) is the same length as \( \overline{DE} \).

**Additional Resources:**

Visions Volume 1, Section 3.3, p. 181
4.1 Real Functions
Question:

In a laboratory, scientists are recording the growth rate of cells. They report that a sample of 50 cells doubled every hour.

Which of the following rules describes the relationship?

A) \( f(x) = 50(x^2) \)

B) \( f(x) = 50(2^x) \)

C) \( f(x) = 50 + 2x \)

D) \( f(x) = 2(50^x) \)

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!
Make a choice!

My Strategies:
Answer and Solution:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (number of hours)</td>
<td>$f(x)$ (total number of cells)</td>
</tr>
<tr>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>$50 \times 2 = 100$</td>
</tr>
<tr>
<td>2</td>
<td>$100 \times 2 = 200$</td>
</tr>
<tr>
<td>3</td>
<td>$200 \times 2 = 400$</td>
</tr>
</tbody>
</table>

An initial amount which increases by the same multiplier is an exponential function

$$f(x) = a(c^x)$$

A) $f(x) = 50(x^2)$
Since we see the term $x^2$, this is a quadratic or second-degree polynomial function.

B) $f(x) = 50(2^x)$
This is an exponential function with an initial value of 50 and a base of 2, which means that it is multiplying by 2 for each increase in $x$.

C) $f(x) = 50 + 2x$
This is a linear function or first-degree polynomial function since we see no exponents.

D) $f(x) = 2(50^x)$
This is an exponential function, but the initial value and the base are switched.

The answer is B.

Specific Strategies:
- Since the choices are different function rules; the goal of the problem is to translate the situation into a functional model.
- Make a table of values to get a clearer picture of the relation
- Doubling the number of cells means you must multiply by 2 for each hour
- Look on your memory aid for the rule of an exponential function

Additional Resources:
- Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
- Visions Volume 2, Section 4.3, p. 39 (Exponential Functions)
Question:

A small town in Quebec already received 120 mm of rain this year when a severe storm occurred. During the storm, rain fell at a constant rate of 5 mm per hour.

The graphs below relate the number of hours since the storm began with the accumulated rainfall in mm.

Which graph below correctly illustrates the relationship?

A)  

B)  

C)  

D)  

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.
Do not leave a blank!
Make a choice!

My Strategies:
QUESTION 69

Answer and Solution:

A) This option is a step function, which means that the amount of rain accumulated remains constant for a period of time and jumps abruptly at critical values; it does not make sense to represent a constant increase of 5mm per hour.

B) This option is a linear function of first-degree polynomial function. Its initial value is 120 and it has a positive slope, indicating a constant increase.

C) This option does not illustrate a constant increase; the amount of rain increases sharply at first, then accumulates more slowly.

D) This option appears to be an exponential function, which would mean that the amount of rain increases by some multiplying factor instead of a constant rate.

The answer is B.

Specific Strategies:

Since the choices are graphs, this is a problem that can be answered by observation; no calculations need to be done.

- Consider all the functional models on your memory aid.
- The linear function model is the only one which offers an interpretation for a constant increase over time.

Additional Resources:

Visions Volume 1, Section 1.2, p. 22 (Lines in the Cartesian Plane)
Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Khan Academy video: http://www.khanacademy.org/math/algebra/algebra-functions/relationships_functions/v/basic-linear-function Ex. Constructing a Function
Question:

The height of a toy hovercraft in relation to the time elapsed since its launch from a table is represented by a piecewise function for the first three minutes of his flight as shown on the graph below.

Which of the following statements is TRUE?

A) The function has no zeros
B) The domain of the function is [0.5, 2].
C) The function is negative over the interval [0.5, 1.5].
D) The function has no extrema.
Answer and Solution:

A) TRUE! The function has no zeros
B) FALSE- The domain is [0, 3]
C) FALSE- The function is positive from [0, 0.5]
D) FALSE- The function has a maximum at (0.5,2)

The answer is A.

Specific Strategies:

Since the answers are properties of functions, we need to analyze the graph to determine the answer. No calculations are necessary.

- Find the definitions for properties of functions on your memory aid
- Make sure you know which properties concern x values and which properties concern y-values

**x-values:** domain, x-intercept
intervals of increase and
decrease, positive and negative
intervals

**y-values:**
range, y-intercept, extrema
(maximum and minimum)

**Zeros:**
A value for x that produces a
value of 0 for y
Or the x-value when line passes
x-axis

**Extrema:**
A point which has the highest
or lowest value of the function.

Additional Resources:

Visions Volume 2, Section 4, (Revision), pp. 7-8 (Properties of Functions)
Khan Academy video: Comparing features of functions 2 (example 1) ; Interpreting features of functions 2 (example 1) ; When a function is positive or negative
Khan Academy video: http://www.khanacademy.org/math/trigonometry/functions_and_graphs/
analyzing_functions/v/when-a-function-is-positive-or-negative
Question: Which of the rules below corresponds to the following graph?

A) \( f(x) = 4x + 3 \)

B) \( f(x) = \left(\frac{4}{3}\right)^x \)

C) \( f(x) = 4x^2 + 3 \)

D) \( f(x) = \frac{43}{x} \)

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank! Make a choice!

My Strategies:
A) The x only has a coefficient so this is a linear function and would be a line.

B) **Correct** - The x is an exponent so this is an exponential function and would curve up (or down) sharply and cross the y-axis.

C) The x is squared so this is a quadratic function and would be a U-shape.

D) The x is a denominator so this is an inverse (rational) function and, in this particular case, would not cross the y-axis (since x≠0).

The answer is B.

**Additional Resources:**

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Visions Volume 2, Section 4.3, p. 39 (Exponential Functions)
Question:

A ball is thrown from a cliff and follows a parabolic trajectory. The function shown below represents the height of the ball according to the time elapsed since it is thrown.

What is the range of the function and what does it mean in concrete terms?

The range of the function is __________. It means ______________

________________________________________________________
**Answer and Solution:**

The range is the set of all possible values of \( y \) from least to greatest (bottom to top).

The lowest value on the graph is \( y = 0 \) and the highest value on the graph is \( y = 25 \).

Therefore the range is \([0, 25]\).

The range of the function is \([0, 25]\). It means the ball will reach its highest point at 25 m and its lowest point at 0 m, or in other words, it will hit the ground.

**Specific Strategies:**

Look for the lowest value of \( y \) on the graph and the highest value of \( y \) on the graph.

Use interval notation in providing your answer. (Remember that the smallest value comes first.)

**Additional Resources:**

Visions Volume 2, Section 4 (Revision), p. 7 (Properties of Functions)
**Question:**

A yard and garden care contractor has developed a mathematical model to determine the price he will charge his clients throughout the season. In order to get his clients interested in his service, he gradually increases his price per hour as the hours accumulate.

He illustrates this model in the graph below.

![Graph showing the price function over hours]

The first piece of the function is a second-degree polynomial function given by the following rule:

\[ g(x) = 10x^2 \quad \text{where} \quad 0 \leq x \leq 8 \]

The price will remain constant for the next 4 hours but after 12 hours, the contractor charges a flat rate of $250 for every four hours of work or part thereof.

One client is charged $1640.

What are the possible numbers of hours that job would have taken?

That job would have taken between _____ and ____ hours.

---

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**
Answer and Solution:

Determine the y-value when \( x = 8 \) for the function \( g(x) \)

\[
g(8) = 10(8)^2
\]

BEDMAS: Remember to apply the exponent first

\[
g(8) = 10 \times 64
\]

\[
= 640
\]

So, the step function begins at a y-value of \$640\ after 8 hours of work. The price doesn’t go up until 12 hours.

<table>
<thead>
<tr>
<th>( x ) (Number of hours)</th>
<th>( y ) (Cost $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[8, 12]</td>
<td>$640</td>
</tr>
<tr>
<td>[12, 16]</td>
<td>$640 + 250 = $890</td>
</tr>
<tr>
<td>[16, 20]</td>
<td>$890 + 250 = $1140</td>
</tr>
<tr>
<td>[20, 24]</td>
<td>$1140 + 250 = 1390</td>
</tr>
<tr>
<td>[24, 28]</td>
<td>$1390 + 250 = $1640</td>
</tr>
</tbody>
</table>

For $1640, it will take 24 to 28 hours, not including 28 hours.

Specific Strategies:

- The first unknown that needs to be determined is the cost for 8 hours of work
- Make sure to keep your work organized from this point – a table is a great idea
- According to the step function, the open circle at \( x = 12 \) is a critical value, so the cost will “jump” starting at 12 hours

Additional Resources:

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Visions Volume 2, Section 4.2, p. 28 (Second-degree Polynomial Function)
Visions Volume 2, Section 4.3, p. 53 (Piecewise Function)
Step Graphs: [http://www.youtube.com/watch?v=LUshzsvoGZU](http://www.youtube.com/watch?v=LUshzsvoGZU)
4.2 Second-Degree Polynomial Function
Question:

Consider the following function:

\[ f(x) = 2x^2 \]

Which of the following graphs represents the function?

A)  
B)  
C)  
D)  

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.

Do not leave a blank!
Make a choice!

My Strategies:
### Answer and Solution:

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>This parabola opens upward, so it must have a positive “a” value. Also, when ( x = 1 ) we can check the y-value by using the rule: ( f(x) = 2(1)^2 = 2 ). Since the parabola appears to pass through the point (1, 2), we can conclude that this is the correct graph.</td>
</tr>
<tr>
<td>B)</td>
<td>This parabola opens upward, so it must have a positive “a”. However, when ( x = 1 ), the value of the function is clearly less than 1, which does not satisfy the equation ( f(x) = 2x^2 ).</td>
</tr>
<tr>
<td>C)</td>
<td>This parabola opens downward, so parameter “a” is a negative value.</td>
</tr>
<tr>
<td>D)</td>
<td>This parabola opens downward, so parameter “a” is a negative value.</td>
</tr>
</tbody>
</table>

The answer is A.

### Specific Strategies:

- **Remember:**
  - For a quadratic or second-degree polynomial function, \( y = ax^2 \)

Since parameter “a” is positive, the parabola should open upward.

Since parameter “a” is longer than 1, the parabola should be narrower than the basic function \( f(x) = x^2 \).

### Additional Resources:

- **Visions Volume 2**, pp. 28-29
- [http://www.purplemath.com/modules/grphquad.htm](http://www.purplemath.com/modules/grphquad.htm)
- Explore Learning Gizmos, [http://www.explorelearning.com/](http://www.explorelearning.com/) look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)
Question:
Which rule represents the following graph?

A) \( y = 0.25x^2 \)
B) \( y = -0.25x^2 \)
C) \( y = -0.25^x \)
D) \( y = 0.25x + 1 \)

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Look carefully at each choice shown (A, then B, then C and then D).
7. Eliminate options you know to be incorrect.
8. Solve/check each possible choice.
9. Select the choice that makes the most sense.
Do not leave a blank!
Make a choice!

My Strategies:
Answer and Solution:

Just by looking... only B matches the graph

A) quadratic opening up
B) quadratic opening down
C) exponential function
D) linear function

Confirm by testing the point...

A) \(-4 = 0.25(4)^2\)
   \(-4 = 4.0\) false

B) \(-4 = -0.25(4)^2\)
   \(-4 = -4.0\) true

C) \(-4 = -0.25^4\)
   \(-4 = 0.0039\) false

D) \(-4 = 0.25(4) + 1\)
   \(-4 = 2.0\) false

Since only one statement is true, that must be the answer.

The answer is B (a quadratic with a negative parameter “a”).

Suggested Strategies:

Imagine the shape of each type of function.

- A quadratic \((ax^2)\) looks like a U.
- An exponential \((ac^x)\) is a curve that either increases or decreases over the whole domain.
- A linear \((ax + b)\) is a straight line.

Recall for a quadratic function if:

- Parameter “a” is positive the parabola opens upward \(\rightarrow\) U is up- Happy Face!
- Parameter “a” is negative the parabola opens downward \(\rightarrow\) U is down- Sad Face!

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video: https://www.khanacademy.org/math/algebra/quadratics/solving graphing_quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, http://www.explorelarning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)
Question:
Consider the following table of values for a quadratic function.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>7.5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>67.5</td>
</tr>
<tr>
<td>20</td>
<td>120</td>
</tr>
</tbody>
</table>

Which of the following rules represents the quadratic function?

A) \( f(x) = -3x^2 \)
B) \( f(x) = -0.3x^2 \)
C) \( f(x) = 0.3x^2 \)
D) \( f(x) = 3x^2 \)
Answer and Solution:

A) \( f(x) = -3x^2 \)
This function opens down (decreases after the vertex) but the table of values increases after the vertex

B) \( f(x) = -0.3x^2 \)
This function opens down (decreases after the vertex) but the table of values increases after the vertex

C) \( f(x) = 0.3x^2 \)
This function opens up and when a non-zero x-value is tested, it gives the corresponding value for \( f(x) \):

\[
f(5) = 0.3(5)^2 = 0.3 \times 25 = 7.5
\]

D) \( f(x) = 3x^2 \)
This function opens up but when a non-zero x-value is tested, it doesn’t give the corresponding \( f(x) \).

\[
f(5) = 3(5)^2 = 3 \times 25 = 75 \neq 7.5
\]

OR

Calculate parameter “a” algebraically using one of the points given:

\[
f(x) = ax^2
\]

for example: \((5, 7.5)\)

\[
\begin{align*}
7.5 &= a(5)^2 \\
7.5 &= a \times 25 \\
\frac{7.5}{25} &= a \\
0.3 &= a
\end{align*}
\]

The answer is C.

Suggested Strategies:

- See what patterns you notice among the data.
  - Symmetry around \((0,0)\)
  - \((0, 0)\) is a minimum so the “a” value will be positive.
  - Since you are told it is quadratic, you know it’s in the form \(ax^2\) so you can calculate \(a\).

Test the two positive functions to see which one works.

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video: https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)
Question:

Point P (5, 10) is on the curve of the 2nd degree function below.

What is the rule of the function?

The rule of the function is ___________________.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
**Answer and Solution:**

The rule for a second degree function is the form $f(x) = ax^2$

Use the coordinates $(5, 10)$ in the function form to determine parameter “$a$”.

\[
f(x) = ax^2 \\
(10) = a(5)^2 \\
10 = 25a \\
\frac{10}{25} = \frac{25a}{25} \\
\frac{2}{5} \text{ or } 0.4 = a \\
f(x) = 0.4x^2 
\]

or $f(x) = 0.4x^2$

The rule of the function is $f(x) = 0.4x^2$

**Specific Strategies:**

Recall the rule for the second degree function with vertex at the origin is $y = ax^2$.

Remember that in any algebraic equation, if you substitute known values, you can solve for the unknown remaining.

**ASK YOURSELF:**

What information is given in the graph? An $(x, y)$ point is given in the graph. If you substitute $x$ & $y$, then only parameter “$a$” will remain to be determined.

**Additional Resources:**

Visions Volume 2, pp. 28-29
Explore Learning Gizmos, [http://www.explorelarning.com/](http://www.explorelarning.com/) look up:

- Quadratic Functions
- Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)
Question:

Match the second degree functions to their respective graphs.

**Function** | **Graph**
---|---
A) $y = 5x^2$ |  
B) $y = -0.2x^2$ |  
C) $y = x^2$ |  
D) $y = -x^2$ |  

**General Strategies:**
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**
Answer and Solution:

<table>
<thead>
<tr>
<th>Function</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>E) $y = 5x^2$</td>
<td>$g$</td>
</tr>
<tr>
<td>F) $y = -0.2x^2$</td>
<td>$h$</td>
</tr>
<tr>
<td>G) $y = x^2$</td>
<td>$f$</td>
</tr>
<tr>
<td>H) $y = -x^2$</td>
<td>$k$</td>
</tr>
</tbody>
</table>

Suggested Strategies:
- Remember what effect parameter “a” has on the curve:
  - Positive opens up
  - Negative opens down
  - The larger the absolute value of “a” the narrower the curve

Additional Resources:
- Visions Volume 2, Section 4.2, pp. 28-29
- Explore Learning Gizmos, [http://www.explorelearning.com/](http://www.explorelearning.com/) look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)
Question:

Gordon is responsible for repairing the soccer field. He needs to purchase a square piece of turf that measures 22.5 m by 22.5 m. He finds the following 2 deals from two different companies:

Company A:

The turf is sold in square pieces and the price is calculated according to its area.

Examples of Cost Based on the Rule Using Length of Side

<table>
<thead>
<tr>
<th>Side length of turf piece</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 m</td>
<td>$1800</td>
</tr>
<tr>
<td>17 m</td>
<td>$5202</td>
</tr>
<tr>
<td>25 m</td>
<td>$11250</td>
</tr>
</tbody>
</table>

Company B:

The turf is also sold by area but the pieces are not necessarily square.

Gordon will buy the piece of turf from the company with the lowest price.

How much will Gordon pay for the piece of turf?

General Strategies:

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer statement.
10. Show any or all your work! Do not leave a blank page!

My Strategies:
Answer and Solution:

Company A:

You can use any of the points to substitute them in \( y = ax^2 \)

\[
\begin{align*}
(1800) &= a(10)^2 \\
1800 &= 100a \\
a &= 18
\end{align*}
\]

\( a = 18 \)

\textit{rule: } y = 18x^2

Now that you have the rule, you can substitute accordingly:

Turf piece of 22.5m (x value):
\[ y=18 \times (22.5)^2 \]
\[ y=9112.5 \]
\[ \$9,112.50 \]

Company B:

Calculate the area of the square piece of turf:

\[ 22.5 \times 22.5 = 506.25 \text{ m}^2 \]

Reading the graph, you can see that 506.25 m\(^2\) corresponds to a cost of \$11,000.

Gordon will buy the turf at the lowest price and therefore he will buy from Company A.

Gordon will pay \$9,112.50 for the turf.

Suggested Strategies:

Company A:

Turf is sold in SQUARE pieces and the cost depends on the AREA \( (s^2) \)

\textbf{ASK YOURSELF:}

Which kind of function is associated with squaring a number? \( y = ax^2 \)

Substitute \((x, y)\) value and solve for “a”. This will give you your rule and you can then substitute and solve for \( y \).

Company B:

Before you can use this graph, you need to calculate the area of the turf Gordon needs.

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video: https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, \texttt{http://www.explorelarning.com/} look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)
4.3 Exponential Function
Question:

A house, initially valued at $275,000, increases in value by 2% annually.

Let:

\[ x \]: represent the number of years and
\[ f(x) \]: represent the value of the house,

Which of the following equations defines this situation?

A) \[ f(x) = 275\,000 \times (0.02)^x \]

B) \[ f(x) = 275\,000 \times (1.02)^x \]

C) \[ f(x) = 275\,000 \times (1.2)^x \]

D) \[ f(x) = 275\,000 \times (0.98)^x \]

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer - what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
**Answer and Solution:**

Exponential Function:
\[ f(x) = a(c)^x \]
where:  
- \( a \) is the initial value and  
- \( c \) is the base

Given the value increases, \( C > 1 \)

2/100 = 0.02 and then adding it to 1

\[ C = 100\% + 2\% = 102\% = 1.02 \]

The correct equation is \( f(x) = 275\,000 \times (1.02)^x \)

A) \( f(x) = 275\,000 \times (0.02)^x \) This would be a decreasing function since the base is less than 1.

B) \( f(x) = 275\,000 \times (1.02)^x \) This is correct.

C) \( f(x) = 275\,000 \times (1.2)^x \) This would represent a growth rate of 20% not 2%.

D) \( f(x) = 275\,000 \times (0.98)^x \) This would represent a decay of 2% (decreasing).

The answer is B.

**Additional Resources:**

Visions Volume 1, pp. 39-41
Question:

Cheryl bought a bike in 2010. Since then, the value of the bike has decreased.

Function $f$ described below represents the value of Cheryl’s bike.

$$f(x) = 505 \times (0.94)^x$$

where $x$ : time elapsed, in years, since Cheryl bought the bike

$f(x)$ : the value of the bike, in dollars

Which of the following statements is true?

A) The initial value is 0.94.

B) The bike’s value decreases by 94% yearly.

C) The graph of the function of the line is linear,

D) In the year 2020, the value of the bike will be $272.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer-what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

The initial value is 505, not 0.94.

The bike’s value decreases by 6%, not 94%.

This is a decreasing function because 0.94 is less than 1.

The value of the bike will be $272 in the year 2020.

Proof:
2020 – 2010 = 10 years
\( f(10) = 505(0.94)^{10} = 272 \)

A) The initial value is 0.94.
   FALSE – the initial value is $505.

B) The bike decreases by 94% yearly.
   FALSE – the rate at which it decreases is 1 – 0.94 = 0.06, or 6%

C) The graph is an increasing function.
   FALSE – since the base is less than one, it is a decreasing function.

D) In the year 2020, the value of the bike will be $272. TRUE!

   2000 – 1990 = 10 years
   \( f(10) = 505(0.94)^{10} = 272 \)

The answer is D.

Suggested Strategies:

- Determine what the initial value is and compare with A).
- Determine the percentage decrease and compare with B).
- Determine if the function is increasing and compare with C).
- Calculate the number of years from 2010 to 2020 and replace value in \( x \) and compare with D).

Additional Resources:

Visions Volume 1, pp. 39-41
<table>
<thead>
<tr>
<th>Question:</th>
</tr>
</thead>
</table>
| The value of a video game depreciates 35% yearly. In 5 years, the price of the video game will be $10.21.  
What is the initial price of the video game? |

<table>
<thead>
<tr>
<th>General Strategies:</th>
</tr>
</thead>
</table>
| 1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Make a prediction about the answer— what will it look like? (an equation, a number, etc.).  
6. Refer to your memory aid, as needed.  
7. Solve.  
8. Ask yourself whether your answer makes sense.  
9. Write your answer. Do not leave a blank! |

<table>
<thead>
<tr>
<th>My Strategies:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The initial price of video game is ___________.</td>
</tr>
</tbody>
</table>
Answer and Solution:

\[ f(x) = ac^x \]

\[ c = 1 - 0.35 = 0.65 \]

\[ f(x) = a (0.65)^x \]

\[ 10.21 = a (0.65)^5 \]

\[ \frac{10.21}{0.1160} = 0.1160 a \]

\[ a = \$88.02 \]

Answer: The initial price of the video game is $88.02.

Suggested Strategies:

- Notice that this is an exponential function that is decreasing.
- Write the equation for this function.
- Calculate the value of “c” which uses the percentage.
- Fill in x with the number of years and y with the price after 5 years.
- Work backwards to find \( a \), the initial price.
- Remember that in decay or decreasing situations we subtract from 100%.

Additional Resources:

Visions Volume 1, pp. 39-41
**Question:**

The function $f$ described below represents the number of bacteria in a well, in relation to the amount of time elapsed since 2005.

$$f(x) = 4500 \cdot (1.33)^x$$

where:

- $x$ is the number of years elapsed since 2005
- $f(x)$ is the number of bacteria

In what year will the number of bacteria be 137,858?

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer.
Do not leave a blank!

**My Strategies:**

In ______ the number of bacteria will be 137,858.
Answer and Solution:

\[
137858 = 4500 \times (1.33)^x
\]

OR

<table>
<thead>
<tr>
<th>x</th>
<th>(4500(1.33)^x)</th>
<th>(f(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(4500(1.33)^1)</td>
<td>5985.0</td>
</tr>
<tr>
<td>2</td>
<td>(4500(1.33)^2)</td>
<td>7960.1</td>
</tr>
<tr>
<td>3</td>
<td>(4500(1.33)^3)</td>
<td>10586.9</td>
</tr>
<tr>
<td>4</td>
<td>(4500(1.33)^4)</td>
<td>14080.5</td>
</tr>
<tr>
<td>5</td>
<td>(4500(1.33)^5)</td>
<td>18727.1</td>
</tr>
<tr>
<td>6</td>
<td>(4500(1.33)^6)</td>
<td>24907.1</td>
</tr>
<tr>
<td>7</td>
<td>(4500(1.33)^7)</td>
<td>33126.4</td>
</tr>
<tr>
<td>8</td>
<td>(4500(1.33)^8)</td>
<td>44058.1</td>
</tr>
<tr>
<td>9</td>
<td>(4500(1.33)^9)</td>
<td>58597.3</td>
</tr>
<tr>
<td>10</td>
<td>(4500(1.33)^{10})</td>
<td>77934.4</td>
</tr>
<tr>
<td>11</td>
<td>(4500(1.33)^{11})</td>
<td>103652.7</td>
</tr>
<tr>
<td>12</td>
<td>(4500(1.33)^{12})</td>
<td>137858.1</td>
</tr>
<tr>
<td>13</td>
<td>(4500(1.33)^{13})</td>
<td>183351.2</td>
</tr>
</tbody>
</table>

\(x = 12\) years later

\[
2005 + 12 = 2017
\]

Answer: In 2017 the number of bacteria will be 137858.

Suggested Strategies:

- Replace \(y\) by the number of bacteria given.
- Work backwards to find the value of \(x\) by guess and check.

OR

- Set up a table of values for the function and find the \(y\) value you are looking for.
- In the table, you can jump ahead to where you think \(x\) would work.
- Your table should contain more than two calculations in order to show evidence of your thinking.

Additional Resources:

Visions Volume 1, pp. 39-41
**Question:**

Sophia invested $5000 today. Her investment will increase by 2.5% each year.

How much profit will she have made in 10 years?

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

Sophia will have made ___________ profit in 10 years.
**Answer and Solution:**

\[ f(x) = a c^x \]

- \( a = \text{initial value} = 5000 \) (principal)
- \( c = \text{growth rate} = 1 + 0.025 = 1.025 \)

OR 100% + 2.5% = 102.5% or 1.025

\[ f(x) = 5000 (1.025)^x \]

where:
- \( x \) is the number of years and
- \( f(x) \) is the total of principal and interest

\[ f(10) = 5000 (1.025)^{10} \]

\[ f(x) = 6400.42 \]

\[ 6400.42 - 5000 = 1400.42 \]

**Answer:** Sophia will have made $1,400.42 profit in 10 years.

**Suggested Strategies:**

- Notice this is an exponential function and that it is increasing so the base is greater than 1.
- Write the equation associated with the function.
- Substitute the initial value for “\( a \)”.
- Determine “\( c \)” by using the percentage.
- Recognize that \( x \) represents the number of years and \( y \) represents the total amount.
- Plug in number of years for “\( x \)”.

**Additional Resources:**

Visions Volume 1, pp. 39-41
Question:

Amy and Ben have deposited money in different banks.

Amy initially deposited $400 in the bank, and deposits $10 into her account every month. No interest is earned.

Ben made a one-time investment of $850 at a yearly interest rate of 4%.

Who will have more money saved after 5 years?

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer-what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:

______________ will have more money after 5 years.
Answer and Solution:

Amy:
\[ f(x) = 10x + 400 \]
where:
- \( x \) is the number of months
- \( f(x) \) is the total amount of money in the bank
After 5 years (5 \times 12 = 60 months), the total amount is:
\[ f(60) = 10(60) + 400 = $1000 \]

Ben:
\[ f(x) = 850(1.04)^x \]
where:
- \( x \) is the number of years
- \( f(x) \) is the total amount of money in the bank
After 5 years, the total amount is:
\[ f(5) = 850(1.04)^5 = $1034.15 \]

The person who will have the most money after 5 years is Ben.

Answer: Ben will have more money after 5 years.

Suggested Strategies:
- Find the function representing Amy & Ben’s situation.
- If the function is linear then “\( a \)” is the amount placed monthly and “\( b \)” is the initial amount invested.
- If the function is exponential then “\( a \)” is the amount invested and “\( c \)” is calculated by using the percentage given.
- Replace \( x \) in both equations by the amount of years given.

Additional Resources:

Visions Volume 1, pp. 39-41
The population of the new city will be __________.
**Answer and Solution:**

Town A: constant function
\[ f(x) = 5000, \]
where:
- \( x \) is the number of years elapsed since 1960
- \( f(x) \) is the total inhabitants.

Town B: exponential function
2020 − 2001 = 19 years elapsed.
\[ f(19) = 2000 \times (1.022)^{19} = 3024 \text{ inhabitants} \]

Town C: linear function
2020 − 2010 = 10 years elapsed
\[ f(x) = 5000 − 50 \times (x) \]
\[ f(19) = 5000 − 50 \times (19) = 4050 \text{ inhabitants}. \]

Town D: exponential function
For \( c = 100\% + 5\% = 105\% = 1.05 \)
- \( a = \) initial population of 1500
\[ f(x) = ac^x \]
\[ f(x) = 1500 \times (1.05)^x \]
\( x = 2020 − 2006 = 14 \)
\[ f(14) = 1500 \times (1.05)^{14} = 2969 \text{ inhabitants} \]

Total:
5000 + 3024 + 4050 + 2969 = 15 043 inhabitants

**Answer:** The population of the new city will be 15 043.

**Additional Resources:**

Visions Volume 1, pp. 39-41
4.4 Step, Periodic and Piecewise Functions
Question:

Every day, a paddle pushes water to create the waves in a wave pool.

The periodic function represented below can be used to determine the position of the paddle from the motor in relation to the time elapsed from the moment the motor is turned on at the beginning of the day.

Which of the following statements is true?

A) \( f(22) = 10 \)

B) The period of this function is 4.

C) The range of this function is \([0, +\infty[\).

D) Over the interval \([6, 8]\), this function is negative.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
**Answer and Solution:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>FALSE - $f(22) = 30$</td>
</tr>
<tr>
<td>B)</td>
<td>TRUE</td>
</tr>
<tr>
<td>C)</td>
<td>FALSE - This is the answer you get for domain.</td>
</tr>
<tr>
<td>D)</td>
<td>FALSE - it is decreasing but still positive since it is above x-axis</td>
</tr>
</tbody>
</table>

**Suggested Strategies:**

To find the period of a periodic function means that the $y$ values will repeat over a certain interval of $x$.

The period can also be thought of as the length of one cycle.

In this case, the graph starts at 10, goes to 30, and back down to 10. At this point the graph starts repeating. Look at the $x$ values to find the period. In this case the graph repeats after 8 units/squares but each unit is worth 0.5 so the period is 4 units.

Do not confuse a negative function with a decreasing one.

**The answer is B.**

**Additional Resources:**

Visions Volume 2, Section 4.4, p. 53
Question:

A store offers a discount of $5 for every $50 in purchases. The graph below illustrates the relation between the value of the purchases and the amount of discount a customer receives.

Consider the following five statements regarding the graph.

1. A customer who spends $150 will receive a $10 discount.
2. A customer who spends $75 will receive a $5 discount.
3. A customer will receive a $5 discount when spending less than $100.
4. A customer will receive twice as much of a discount when spending $200 than $100.
5. A customer will receive no discount when spending less than $50.

Which of the statements above are true?

A) 2, 4 and 5
B) 2, 3 and 4
C) 1, 2 and 4
D) 1, 2 and 3

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:
Read each statement carefully.

1. A customer that spends $150 will receive a $10 discount.
   
   \[ \text{False} - 150=15 \text{ discount} \]

2. A customer that spends $75 will receive a $5 discount.
   
   \[ \text{True} - \text{In values between the endpoints of the step has the same } y\text{-value.} \]

3. A customer will receive a $5 discount when spending less than $100.
   
   \[ \text{False} - \text{Be careful with the less than } 100. \text{ Less than } 100 \text{ includes less than } 50. \text{ Less than } 50 \text{ equals } 0 \text{ discount.} \]

4. A customer will receive twice as much of a discount when spending $200 than $100.
   
   \[ \text{True} - \text{The discount for 200 = 20 and 100= 10} \]

5. A customer will receive no discount when spending less than $50.
   
   \[ \text{True} - \text{Less than 50 is the step on the x-axis.} \]

The answer is A.

Additional Resources:
Visions Volume 2, Section 4.4, p. 53
Question:

The cost to park a car in a particularly expensive lot is $40 for the first half hour and $5.00 for each additional hour or part thereof.

A customer uses this parking lot for five hours.

How much will the customer pay for parking?

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:

The customer will pay $________ for parking 5 hours.
Answer and Solution:

Making a table to show how the cost changes over time.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>]0,0.5]</td>
<td>$40</td>
</tr>
<tr>
<td>]0.5,1.5]</td>
<td>$45</td>
</tr>
<tr>
<td>]1.5,2.5]</td>
<td>$50</td>
</tr>
<tr>
<td>]2.5,3.5]</td>
<td>$55</td>
</tr>
<tr>
<td>]3.5,4.5]</td>
<td>$60</td>
</tr>
<tr>
<td>]4.5,5.5]</td>
<td>$65</td>
</tr>
<tr>
<td>]5.5,6.5]</td>
<td>$70</td>
</tr>
</tbody>
</table>

The question asks for 5 hours which is located in the interval ]4.5,5.5] at a cost of $65.

Notice the use of intervals. The interval ]0, 0.5] indicates the hours ranging from anything above 0 up to and including 0.5 hours. Although the customer parks a car for ¼ of an hour, they will be charged $40.

OR

You could also consider the very specific case: $40 for the first half hour plus 4.5 hours → rounded up to 5 hours x $5 per hour: 40 + (5 x 5) = 40 + 25 = 65.

Answer: The customer will pay $65 for parking 5 hours.

Suggested Strategies:

1) Identify that this problem is a step function.

2) Note the key words in the problem. “for each additional hour or part thereof.”

   Take a moment to highlight the key words.

   “part thereof” means that the customer will be charged for $5 even if they did not complete one hour.

3) Recall the use of intervals. Notice the difference in the brackets. ]0, 0.5]

   The bracket here means including 0.5, while the other bracket does not include 0.

Additional Resources:

Visions Volume 2, Section 4.4, p. 53
Question:

Two companies offer different prices for internet service. Company A uses a linear model where each 100 gigabytes of usage will cost $20. Company B follows a greatest integer function as shown on the graph below.

What is the difference in cost between the two companies for 200 gigabytes?

The difference in cost is ___________.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer - what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

Company A:

\[
\frac{100 \text{ gb}}{20} = \frac{200 \text{ gb}}{x} \rightarrow x = 40
\]

OR

We can see the line passing through the origin (0, 0) and the given point (100, 20)

\[
\frac{20 \text{ per 100 gigabytes}}{0.20 \text{ per 1 gigabyte}}
\]

The function rule to calculate the cost in relation to the internet usage (gigabyte) will be \( f(x) = 0.20x \)

The cost of 200 gigabyte \( f(200) \) = $40

Company B:

From the graph – \( f(200) \) = $25
(Note that it isn’t $35 because it’s the solid dot that indicates the y-value.)

The difference: $40 – $25 = $15

Answer: The difference in cost is $15.

Suggested Strategies:

1) Since we can’t read off the graph for the cost of 200 gb from Company A, we need to look further.
   - The text says each 100 gb costs $20 so we could set up a proportion or
   - The graph shows the same information but we can find a unit rate which will be the “\( a \)” in \( y = ax + b \). \( b \) will be 0 since the line goes through the origin.

Recall: ‘\( a \)’ in the equation represents the slope \( \frac{y_2 - y_1}{x_2 - x_1} \) and ‘\( b \)’ represent the y-intercept. (In this case it is 0 since the line passes through the origin.

2) By looking at the graph we can see the exact value of the cost of Company B at 200 gb. Be careful not to take the $35 value. Remember the difference between a white circle and a black circle.

Additional Resources:

Visions Volume 2, Section 4.4, p. 53
A store selling World Cup memorabilia places a mechanical mascot in front of the store.

The mascot raises a ball from the ground to a maximum height of 150 cm at a constant rate, holds it there for 20 seconds, and then lowers it back to ground level at the same rate.

The graph below illustrates a periodic function that represents the height, or the distance between the ball and the ground in relation to the time elapsed in seconds.

A store employee turns on the mechanism that moves the soccer ball at 8:00AM. At that point the ball is at ground level. At exactly 8:15 AM, the mechanism breaks down and the soccer ball stops moving.

How high above the ground is the ball when the mascot stops moving?

The ball is _______ cm off the ground when the mascot stops moving.
**Answer and Solution:**

**Step 1: Determine the period of the function**

5 complete cycles = 400 seconds  
1 complete cycle = 80 seconds

**Step 2: Determine how many complete cycles from 8:00-8:15 AM**

Since our unit of time on our graph is in seconds we need to convert minutes to seconds.

15 minutes x 60 seconds/minute = 900 seconds

Determine the number of complete cycles in 900 seconds

\[
\frac{900 \text{ seconds}}{80 \text{ seconds/cycle}} = 11.25 \text{ cycles}
\]

OR

The graph (5 cycles) covers 400 seconds, so 800 seconds covers the graph twice, leaving 100 seconds (900 – 800) left over to being the third time. From reading the graph, you can see that at 100 seconds, the ball is 100 cm above the ground.

HOWEVER, even though it looks like 100, make sure by showing the following:

**Step 3: Determine the height the ball is relative to the ground at 20 seconds**

To find the exact value we need to break a cycle into pieces.

At 20 seconds the ball will be moving upwards. To know the exact height we will have to find the equation of a line: \( y = ax + b \).

We have two points on the line. \( P_1(0,0) \) and \( P_2(25,125) \)

\[
a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{125 - 0}{25 - 0} = \frac{125}{25} = 5 \quad \text{and} \quad b = 0 \quad \text{since the y-intercept is at the origin.}
\]

The equation of the line is \( y = 5x \)

To find the height of the ball at 20 seconds, substitute \( x=20 \) and solve for \( y \).

\[
y = 5(20) = 100 \text{ cm}
\]

The ball is **100 cm** off the ground when the mascot stops moving.

---

**Additional Resources:**

Visions Volume 2, Section 4.4, p. 53
The graph below represents the outline of a skateboard ramp which corresponds to a piecewise function defined by:

\[ f(x) = \begin{cases} 
ax^2 & \text{if } 0 \leq x \leq 80 \\
256 & \text{if } 80 \leq x \leq 160 \\
-1.25x + b & \text{if } 160 \leq x \leq 320 
\end{cases} \]

For security purposes, a strip of reflective tape will be placed on the ramp at a height of 144 cm.

What is the length of this piece of reflective tape?

The length of the piece of reflective tape is \__________ cm.
Answer and Solution:

Step 1: Solving for the parameter “a” in the second-degree function

Note: The second-degree function is connected to the constant function. To determine the coordinates of the point connecting the two functions we need to look at the interval in the function rule.

When \( x = 80 \), the \( y \)-coordinate is at 256.

Substitute the coordinates into the second-degree equation to solve for ‘\( a \)’

\[
a(80)^2 = 256
\
a = \frac{256}{6400} = \frac{1}{25} \text{ or } 0.04
\]

Step 2: Solving for parameter “b” in the linear equation.

The function connecting the linear function is the constant function.

When \( x = 160, y = 256 \)

\[
256 = -1.25(160) + b
\
256 + 200 = b
\
b = 456
\]

Step 3: Find the x coordinates when \( y = 144 \)

\[
\begin{align*}
\ln f(x) &= 0.04x^2 & \ln f(x) &= -1.25x + 456 \\
144 &= 0.04x^2 & 144 &= -1.25x + 456 \\
3600 &= x^2 & -312 &= -1.25x \\
x &= 60 & x &= 249.6
\end{align*}
\]

Step 4: The length of tape

\[
249.6 \text{ cm} - 60 \text{ cm} = 189.6 \text{ cm}
\]

The length of the piece of reflective tape is 189.6 cm

Additional Resources:

Visions Volume 2 Section 4.4 p. 53
http://www.mathsisfun.com/sets/functions-piecewise.html
http://www.purplemath.com/modules/strtlneq.htm

Suggested Strategies:

1) Identify the type of functions in the piecewise function.

Remember a piecewise function is made up of different functions defined by a certain interval of \( x \)-values (domain).

2) To solve for the missing parameters you will to know the coordinates of the points that connect the functions together

3) Find the \( x \) coordinates of the two extremities of the tape
5.1 Trigonometric Ratios
Question:

Consider the right triangle ABC shown below.

Which of the following expressions represents the correct trigonometric ratio for angle A?

A) \( \sin A = \frac{9}{2} \)

B) \( \tan A = \frac{9}{2} \)

C) \( \cos A = \frac{2}{9} \)

D) \( \tan A = \frac{2}{9} \)
## QUESTION 93

### Answer and Solution:

Since the opposite and adjacent sides are given, the correct trigonometric ratio to select is Tangent (tan).

Tangent is the ratio of \(\text{opposite side of an angle} \div \text{adjacent side of an angle}\)

So, \(\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{2}{9}\)

- A) \(\frac{9}{2}\) would be cot A not sin A
- B) \(9\) would be cot A not tan A
- C) \(\frac{2}{9}\) would be tan A not cos A
- D) correct

**The answer is D.**

### Specific Strategies:

Which two sides are given in relation to angle A?

- **OPPOSITE & ADJACENT**

1. Identify the sides according to angle A.
2. Given the “knowns” (opposite and adjacent), select the correct trigonometric ratio.

### Additional Resources:

Visions Volume 2, p. 84
Khan Academy Video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/basic-trigonometry](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/basic-trigonometry)
Question:
Consider right triangle DEF shown below.

Which trigonometric ratio corresponds to the ratio $\frac{d}{e}$?

A) $\cos D$
B) $\tan D$
C) $\cos F$
D) $\sin F$

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

A) \( \cos D = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{f}{e} \) incorrect

B) \( \tan D = \frac{\text{opposite}}{\text{adjacent}} = \frac{d}{f} \) incorrect

C) \( \cos F = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{d}{e} \) correct

D) \( \sin F = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{f}{e} \) incorrect

The answer is C.

Specific Strategies:

1. Consider each ratio one at a time
2. Label sides according to angle D
3. Label sides according to angle F

Always solve for each choice (A, B, C and D).

Don’t stop when you think you have the correct answer.

Additional Resources:

Visions Volume 2, p. 84
Khan Academy video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/basic-trigonometry
http://www.purplemath.com/modules/basirati.htm
**Question:**

Consider right triangle DEF below.

To the nearest hundredth, what is the length of side DF?

A) 4.82 cm  
B) 10.33 cm  
C) 12.58 cm  
D) 24.45 cm

**General Strategies:**

1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Solve the problem **without looking** at choices shown (A, B, C and D).  
7. Look at all the choices.  
8. Match your answer to the appropriate choice.  

*Do not leave a blank!*  
Make a choice!

**My Strategies:**
**Answer and Solution:**

- Length DF is the opposite side and the unknown (x)
- The hypotenuse is given
- Therefore, the correct trigonometric ratio to use is SIN (SOH)

**MAKE SURE YOUR CALCULATOR IS IN DEGREES**

\[
\sin E = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\sin 65 = \frac{x}{11.4}
\]

\[
x = 11.4 \sin (65^\circ)
\]

\[
x = 10.33
\]

A) This is the result of cos 65° = \(\frac{x}{11.4}\) which is incorrect.

B) 10.33 cm is correct.

C) This is the result of sin 65° = \(\frac{11.4}{x}\) which is incorrect.

D) This is the result of tan 65° = \(\frac{11.4}{x}\) which is incorrect.

The answer is B.

**Specific Strategies:**

- Which angle is given?
- Which side is given?
- Which side is the unknown?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle.
2. Select the appropriate trigonometric ratio (sin, cos or tan).
3. Solve for the unknown.

**Additional Resources:**

Visions Volume 2, p. 84
Khan Academy video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle)
**Question:**

During a football practice, player A passes the ball to player B. The diagram below illustrates the pass.

What is the distance traveled by the ball during the pass?

The distance traveled by the ball during the pass is \[ \text{__________ m.} \]

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**

The distance traveled by the ball during the pass is \[ \text{__________ m.} \]
Answer and Solution:

- Length of the adjacent side is given: 12 m
- The hypotenuse is the unknown ($x$)
- Therefore, the correct trigonometric ratio to use is COS (CAH)

MAKE SURE YOUR CALCULATOR IS IN DEGREES

\[
\cos 22.6^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

\[
0.9232 = \frac{12}{x}
\]

\[
x = \frac{12}{0.9232} = 12.998 \text{ or } 13 \text{ m}
\]

Answer: The distance traveled by the ball during the pass is 13.0 m.

Specific Strategies:

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle.
2. Select the appropriate trigonometric ratio (sin, cos or tan).
3. Solve for the unknown.

Additional Resources:

Visions Volume 2, p. 84; examples on p. 85
Khan Academy video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle
http://www.purplemath.com/modules/basirati2.htm
Question:

In isosceles triangle RST, height QT measures 13.1 cm.

What is the measure of angle R to the nearest tenth?

The measure of angle R is _______________.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

- The focus is on triangle TQR
- Triangle TQR is a right triangle because the height is perpendicular to the base.
- The measurement of QR is $31 \div 2$ because the triangle is isosceles
- Use trigonometric ratios
- The “knowns” are the opposite side and adjacent side to angle R
- The correct trigonometric ratio is Tan (TOA)

*Remember to divide RS by two to obtain the value of the adjacent side of angle R.* $31 \div 2 = 15.5$

\[
\tan R = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan R = \frac{13.1}{15.5}
\]

\[
R = \tan^{-1} 0.8451
\]

\[
R = 40.2^\circ
\]

The measure of angle R is $40.2^\circ$.

Specific Strategies:

Which angle is given? or unknown?

Which sides are given?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle
2. Select the appropriate trigonometric ratio (sin, cos or tan)
3. Solve for the unknown

Additional Resources:

Visions Volume 2, pp. 84-85
Khan Academy video: [https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle](https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle)
**Question:**

In the figure below, triangle BEF and triangle ABC are right triangles.

In addition,

\[
\overline{DB} \text{ is an altitude of right triangle ABC} \\
\overline{AE} \cong \overline{BE} \\
m\angle BEF = 36^\circ \\
m\overline{EF} = 15\text{ cm} \\
m\overline{DB} = 9\text{ cm}
\]

The drawing is not to scale.

What is the area of figure AEFC?

---

**General Strategies:**

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

The area of figure AEFC is ____________ cm².
Answer and Solution:

\[ AB = 2 \times EB \]
\[ = 2 \times 8.82 \]
\[ = 17.64\text{cm} \]

\[ BD \] is an altitude of triangle ABC, so we can use the Pythagorean theorem to find \( AD \):
\[ AD^2 + 9^2 = 17.64^2 \]
\[ AD = 15.17\text{cm} \]

We can also use metric relations:
\[ AB^2 = AD \times AC \]
\[ 17.64^2 = 15.17 \times AC \]
\[ AC = 20.51 \text{ cm} \]

\[ AC \times DB = AB \times BC \]
\[ 20.51 \times 9 = 17.64 \times BC \]
\[ BC = 10.46\text{cm} \]

Area = \[ \frac{base \times height}{2} \]
\[ = \frac{10.46 \times 17.64}{2} = 91.32 \text{ cm}^2 \]

Total area = 53.54 cm\(^2\) + 91.32 cm\(^2\) = 144.86 cm\(^2\)

Specific Strategies:

Break the image down into two right triangles: ABC and BEF. Determine the area of each and then add them.

Formulas that may apply to right triangles:
- Pythagorean theorem
- Trigonometric ratios
- Metric relations

Start with triangle BEF:
Using the sin ratio, determine the length of side BE:
\[ \sin(36^\circ) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BE}{15} \]
\[ 15 \times 0.5878 \approx 8.82\text{ cm} \]

Using the cos ratio (or Pythagorean Theorem), determine the length of side BF:
\[ \cos(36^\circ) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BF}{15} \]
\[ 15 \times 0.8090 \approx 12.14\text{ cm} \]

Determine the area of triangle BEF:
\[ \text{Area} = \frac{\text{base} \times \text{height}}{2} \]
\[ = \frac{12.14 \times 8.82}{2} = 53.54 \text{ cm}^2 \]

Additional Resources

Visions Volume____, Section 3.3, pp. 178-185
Visions Volume____, Section 5.1, pp. 91-101
5.2 Finding Missing Measurements
Question:

A sheet of construction paper is cut along the dashed lines.

What is the angle formed between the two dashed lines?

A) 19.7°
B) 39.4°
C) 70.3°
D) 140.6°

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

\[ \tan x = \frac{28}{10} \]

\[ x = \tan^{-1}(2.8) = 70.3^\circ \]

\[ ? = 180^\circ - 2(70.3^\circ) = 39.4^\circ \]

(You may instead get the result 39.3° which would be correct.)

A) Incorrect. Did \( \tan x = \frac{10}{28} \) and forgot to subtract from 180°.

B) Correct answer.

C) Incorrect. Forgot to subtract from 180°.

D) Incorrect. Did \( \tan x = \frac{10}{28} \) and subtracted from 180°.

The answer is B.

Additional Resources:

Visions Volume 1 p. 95 (Mathematical Knowledge Summary)
Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
Question:

A stationary supplier needs to calculate the angle formed by the flap as shown in the diagram. The envelope is 20 cm long and 7 cm wide. The flap folds over a maximum of 3 cm at its center.

What is the measure of this angle?

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:

A) 16.7°
B) 17.5°
C) 19.3°
D) 20.5°
Answer and Solution:

\[
\tan A = \frac{3}{10}
\]

\[
\angle A = \tan^{-1} \left( \frac{3}{10} \right)
\]

\[
\angle A = 16.7^\circ
\]

A) Correct Answer.

B) Incorrect. Used \( \sin \frac{3}{10} \) instead of \( \tan \frac{3}{10} \).

C) Incorrect. Used \( \tan \frac{7}{20} \) instead of \( \tan \frac{3}{10} \).

D) Incorrect. Used \( \sin \frac{7}{20} \) instead of \( \tan \frac{3}{10} \).

Suggested Strategies:

- Look at the illustration and find the right triangle(s) – you should see the two (implied) right triangles in the obtuse triangle of the flap.
- Since the 3 cm goes down the center, the top can be divided into 10 cm and 10 cm and you have two right triangles.
- (Notice that the 7 cm measure is extra information that you don’t need.)
- Now you’re ready to choose a trig ratio to solve for the missing angle. Tangent is what works in this case.
- Turn your paper around if the orientation of the triangle is confusing.

The answer is A.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions
Question:

Sylvain wants to paint the surface of his triangular deck.

What is the area to be painted?

A) 87.3 m²
B) 93.6 m²
C) 119.4 m²
D) 137.3 m²

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Refer to your memory aid, as needed.
6. Solve the problem without looking at choices shown (A, B, C and D).
7. Look at all the choices.
8. Match your answer to the appropriate choice.

Do not leave a blank! Make a choice!

My Strategies:
Answer and Solution:

C) 119.4 m²

\[ \tan 43° = \frac{h}{16} \]

\[ 0.19325 = \frac{h}{16} \]

\[ h = 16 \times 0.19325 \]

\[ h = 14.92 \text{ m} \]

\[ A = \frac{16 \times 14.92}{2} = 119.4 \text{ m}^2 \]

A) Incorrect. \( \sin 43° \) was used instead of \( \tan 43° \).

B) Incorrect. \( \cos 43° \) was used instead of \( \tan 43° \).

C) Correct answer.

D) Incorrect. \( \tan 47° \) was used instead of \( \tan 43° \).

The answer is C.

Suggested Strategies:

- Finding the area of a triangle means we need either the measure of all three sides (and use Hero’s formula) or we need a base and its height.
- Here we have one side and an angle...in a right triangle so trig ratios can be used to find the measures of missing sides.
- If we consider the 16 m side as the base, the height is opposite the 43° angle.
- Tangent is the trig ratio to use in this case.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
Question:

Bird nests are sitting at the top of two poles. Pole A is 11.5 m long and is leaning at an 8° angle from the vertical; Pole B is 11 m long and is leaning at a 5° angle from the vertical.

What is the difference in height between the two bird nests?

Round your answer to the nearest hundredth.

The difference in height between the two bird nests is __________ m.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:

The difference in height between the two bird nests is __________ m.
**Answer and Solution:**

Difference in height between the bird nests = 0.43 m

\[
\cos 8^\circ = \frac{\text{Height}_A}{11.5} \Rightarrow 0.990 = \frac{\text{Height}_A}{11.5} \Rightarrow \text{Height}_A = 11.39 \text{ m}
\]

\[
\cos 5^\circ = \frac{\text{Height}_B}{11} \Rightarrow 0.996(11) = \text{Height}_B \Rightarrow \text{Height}_B = 10.96 \text{ m}
\]

Difference is 11.39 – 10.96 = 0.43 m

The difference in height between the two bird nests is **0.43 m**.

**Suggested Strategies:**

- Before you can find the difference in the heights of the bird nests, you need to find the height from the ground of each one.
- Height is a vertical distance so the height of each nest will be less than the length of the poles.
- You will have to identify right triangles and their elements before you can use trig ratios to solve for the missing measure.

**Additional Resources:**

- Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
- Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
**Question:**

A flagpole is anchored using two guy wires. The guy wire on the right is 18 m long and has an angle of inclination with the ground of 30°. It is attached 1 m below the point where the left guy wire is attached to the pole. The left guy wire is located 20 m from the base of the flagpole.

What is the angle of inclination of the left guy wire?

The angle of inclination of the left guy wire is \( \text{___________} \).

### General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

### My Strategies:

The angle of inclination of the left guy wire is \( \text{___________} \).
Answer and Solution:

\[
\sin 30^\circ = \frac{x}{18} \\
0.5 = \frac{x}{18} \\
x = 0.5 \times 18 = 9 \text{ m}
\]

or the length of the side opposite a 30° angle in a 30°-60°-90° triangle is ½ the length of the hypotenuse.

\[
y = \tan^{-1}\left(\frac{10}{20}\right) \\
y = \tan^{-1}(0.5) \\
y = 26.6^\circ
\]

The angle of inclination of the left guy wire is 26.6°.

Suggested Strategies:

- Determine what you need to know about the left triangle in order to find the missing angle measure.
  - Either the hypotenuse (the length of the guy wire),
  - the other angle or
  - the height up the flagpole of the other guy wire.
- We only have enough information to find the last option.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
Question:

In triangle PQR, we have $m\angle P = 42^\circ$; $m\overline{PR} = 15\ cm$; $m\overline{QR} = 12\ cm$.

In addition, $\angle PQR$ is an obtuse angle.

To the nearest degree, what is the measure of $\angle PQR$?

The measure of $\angle PQR$ is ____________.

General Strategies:
1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer-what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
Answer and Solution:

Use Sine Law to find $m\angle P\!Q$

\[
\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}
\]

\[
\frac{12 \text{ cm}}{\sin 42^\circ} = \frac{15 \text{ cm}}{\sin Q}
\]

\[
\sin Q = \frac{15 \text{ cm} \cdot \sin 42^\circ}{12 \text{ cm}}
\]

\[
\sin Q = 0.8364132579
\]

Find the inverse of Sine, use

\[
\sin^{-1}(0.8364132579) = 56.733^\circ
\]

Extra step – since we are looking for an obtuse angle...

\[
\sin Q = \sin(180^\circ - Q)
\]

\[
\sin(56.7633^\circ) = \sin(180^\circ - 56.7633^\circ)
\]

\[
180^\circ - 56.7633^\circ = 123.2367^\circ
\]

The measure of $\angle PQR$ is 123°.

Suggested Strategies:

- Since this is not a right triangle, you cannot use SOH CAH TOA
- Don’t forget that when you use Sine Law to determine the measure of an obtuse angle, you must subtract your answer from 180.

Careful, this is not an obtuse angle!!

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
**Question:**

Sally is flying a kite. The tip of the kite is 60 m above the ground and the kite itself is 2 m in length. She is holding the string 1 m above the ground. The angle of inclination of the string started out at 55° but then the wind shifted and the angle of the string decreased to 40°. In order to maintain the height of the kite, Sally had to let more string out from the spool.

How much string did Sally need to let out to maintain the height of the kite?

**General Strategies:**
1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, make an educated guess and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.

Show any or all your work! Do not leave a blank page!

**My Strategies:**

Sally had to let out an additional _______ of string to maintain the height of the kite.
**Answer and Solution:**

Height of kite where the string is attached is 60 m – 2 m = 58 m

Sally is holding the string 1 m up so height for calculation is 58 m – 1 m = 57 m

Length of string originally:
\[
sin 55° = \frac{57 \text{ m}}{l_1}
\]
\[
0.8191 = \frac{57 \text{ m}}{l_1}
\]
\[
l_1 = \frac{57 \text{ m}}{0.8191} = 69.58 \text{ m}
\]

Length of string after the wind shift:
\[
sin 40° = \frac{57 \text{ m}}{l_2}
\]
\[
0.6428 = \frac{57 \text{ m}}{l_2}
\]
\[
l_2 = \frac{57 \text{ m}}{0.6428} = 88.68 \text{ m}
\]

\[
88.68 \text{ m} - 69.58 \text{ m} = 19.1 \text{ m}
\]

Sally had to let out an additional 19.1 m of string to maintain the height of the kite.

**Suggested Strategies:**

- The tricky part of this question is trying to imagine what exactly is happening.
- From the diagram, you can probably see that triangles are involved; sketch them separately in a way that will make it easier to see how to set up the trig ratios
- Recognize that the string is the hypotenuse of these triangles and get the feel that the hypotenuse will be longer if the angle is smaller.
- Now figure out the two lengths and find the difference.

**Additional Resources:**

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
5.3 Calculating the Area of any Triangle
**Question:**

Consider the following diagram of triangle ABC.

What is the measure of angle CAB?

A) 53.1°  
B) 52.4°  
C) 38.7°  
D) 36.8°

**General Strategies:**

1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Solve the problem **without looking** at choices shown (A, B, C and D).  
7. Look at all the choices.  
8. Match your answer to the appropriate choice.  
   Do not leave a blank!  
   Make a choice!

**My Strategies:**
**Answer and Solution:**

\[
\frac{a}{\sin A} = \frac{c}{\sin C}
\]

\[
\frac{12}{\sin A} = \frac{15}{\sin 98°}
\]

\[
12(\sin 98°) = 15\sin A
\]

\[
\sin A = 0.79
\]

\[
A = 52.39°
\]

A) 53.1°; Did not use sine law, used \( \sin CAB = \frac{a}{c} \) instead.

B) **52.4°; Correct Answer.**

C) 38.7°; Did not use sine law, used \( \tan CAB = \frac{a}{c} \) instead.

D) 36.8°; Did not use sine law, used \( \cos CAB = \frac{a}{c} \) instead.

The answer is B.

**Specific Strategies:**

- Notice that this is **not** a right angle (90°) triangle.
- You **cannot** apply SOH, CAH, TOA.
- You should use Sine law.
- Color code, highlight or match the angles with their corresponding sides.
- Apply Sine law formula.

**Additional Resources:**

Visions Volume 2, Section 5.3, p. 108
Question:

Consider the following diagram of triangle ABC. All measurements are in metres.

What is the area of triangle ABC?

A) \(7.35 \text{ m}^2\)  
B) \(25.5 \text{ m}^2\)  
C) \(132.3 \text{ m}^2\)  
D) \(187.1 \text{ m}^2\)

General Strategies:

1. Read the question.  
2. Highlight key words.  
3. Identify the math topic.  
4. Re-read the question.  
5. Refer to your memory aid, as needed.  
6. Solve the problem without looking at choices shown (A, B, C and D).  
7. Look at all the choices.  
8. Match your answer to the appropriate choice.  
Do not leave a blank! Make a choice!

My Strategies:
QUESTION 107

Answer and Solution:

\[
p = \frac{a + b + c}{2}
\]

\[
p = \frac{(21 + 18 + 15)}{2} = 27
\]

\[
A = \sqrt{p(p-a)(p-b)(p-c)}
\]

\[
A = \sqrt{27(27-21)(27-18)(27-15)}
\]

\[
A = \sqrt{27(6)(9)(12)}
\]

\[
A = \sqrt{17 496}
\]

\[
A = 132.3 \text{ m}^2
\]

Area = 132.3 m²

A) 7.35 m²; added the values under the radical

B) 25.5 m²; Forgot to multiply everything by p under the radical.

C) 132.3 m²; Correct Answer

D) 187.1 m²; Multiplied everything by the perimeter instead of the half perimeter

The answer is C.

Specific Strategies:

Identify the values of \(a\), \(b\) and \(c\) according to the diagram provided.

Calculate the value of the half perimeter.

Substitute the values for \(a\), \(b\), \(c\), and \(p\) in Heron’s formula.

Additional Resources:

Visions Volume 2, Section 5.3, p. 108
Khan Academy video: https://www.khanacademy.org/math/geometry/basic-geometry/heron_formulatutorial/v/heron-s-formula Heron’s Formula
**Question:**

Consider triangle ABC shown below.

To the nearest degree, what is the measure of angle $AB$?

The measure of angle $ABC$ is ____________ .

**General Strategies:**

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

**My Strategies:**
## Answer and Solution:

Knowing that angle C is 55°, that side C has a length of 10 and side b a length of 7, we can calculate the size of angle B using sine law.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

To solve the problem, we need three of the four values in any given equality. Since we know b, c and \(\sin C\), then:

\[
\frac{b}{\sin B} = \frac{c}{\sin C}
\]

Rearranging the equation we get:

\[
\sin B = \frac{bsinC}{c}
\]

Substituting the corresponding values, we get:

\[
\sin B = \frac{7\sin 55^\circ}{10}
\]

\[
\sin B = 0.573406431
\]

Finding the inverse of sin, \(\sin^{-1}\), we can get the measure of the angle:

\[
B = \sin^{-1} 0.573406431
\]

\[
B = 35^\circ
\]

**Answer:** Angle B = 35°

## Additional Resources:

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)

Question:

What is the area of triangle ABC shown below?
All measurements are in cm.

The area of triangle ABC is ____________.

General Strategies:

1. Read the question.
2. Highlight key words.
3. Identify the math topic.
4. Re-read the question.
5. Make a prediction about the answer—what will it look like? (an equation, a number, etc.).
6. Refer to your memory aid, as needed.
7. Solve.
8. Ask yourself whether your answer makes sense.
9. Write your answer. Do not leave a blank!

My Strategies:
**Answer and Solution:**

Before starting with Hero’s formula, we must first determine the half-perimeter \( p \) of the triangle.

\[
\text{Since } p = \frac{(a + b + c)}{2}
\]

where \( a = 7 \text{ cm}, b = 5 \text{ cm} \) and \( c = 6 \text{ cm} \)

we know that \( p = \frac{(7 \text{ cm} + 5 \text{ cm} + 6 \text{ cm})}{2} = 9 \text{ cm} \)

Now using the formula we get:

\[
A = \sqrt{p(p - a)(p - b)(p - c)}
\]

\[
A = \sqrt{9(9 - 7)(9 - 5)(9 - 6)}
\]

\[
A = \sqrt{9(2)(4)(3)}
\]

\[
A = \sqrt{216}
\]

\[
A = 14.7 \text{ cm}^2
\]

Area = 14.7 cm\(^2\)

The answer is 14.7 cm\(^2\).

**Specific Strategies:**

Identify the values of \( a, b \) and \( c \) according to the diagram provided.

Calculate the value of the half perimeter.

Substitute the values for \( a, b, c, \) and \( p \) in Hero’s formula.

**Additional Resources:**

Visions Volume 2 Activity 2 p. 105 and Mathematical Knowledge p. 108

Khan Academy video: [http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry](http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry) Sine, Cosine and Tangent Trigonometric Functions
Question:

Consider triangle ABC shown below. What is the length of segment AB? Round your answer to the nearest tenth.

The length of segment AB is ________________ .
Answer and Solution:

Knowing that angle B is 53°, that side b has a length of 6 and angle C is 70°, we can calculate the length of side c using sine law.

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

To solve the problem, we need three of the four values in any given equality. Since we know b, \( \sin B \) and \( \sin C \), then:

\[ \frac{b}{\sin B} = \frac{c}{\sin C} \]

Rearranging the equation we get:

\[ c = \frac{bsinC}{sinB} \]

Substituting the corresponding values, we get:

\[ c = \frac{6\sin70°}{\sin53°} \]

\[ c = 7.06 \]

Since side c corresponds to AB, the length of segment AB is 7.06.

Answer: the length of segment AB is 7.06.

Specific Strategies:

- Notice that this is not a right angle (90°) triangle
- You cannot apply SOH, CAH, TOA
- You should use Sine law
- Color code, highlight or match the angles with their corresponding sides
- Apply Sine law formula

Additional Resources:

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry Sine, cosine and tangent trigonometric functions
The Space Needle is a tall structure in Seattle, Washington. Phil, a math student, attempts to estimate the height of the Space needle by using a clinometer, a device that measures the angle of inclination.

First, Phil stands at point C and reads a $50^\circ$ angle on the clinometer. Then, Phil moves 353 m to Point B and reads an angle of $20^\circ$ on the clinometer. Phil estimates the Space Needle is between 182 m and 188 m in height.

Based on the information given, is Phil’s estimation correct? Explain.

☐ Yes, his estimation is correct.

☐ No, his estimation is not correct.

General Strategies:

1. Read the problem.
2. Highlight key words.
3. Identify the math topics.
4. Re-read the problem.
5. Define your steps (your game plan) – this is criteria 3.
6. Refer to your memory aid, as needed.
7. Solve.
8. If you get stuck on a calculation, pick a number and keep going.
9. Ask yourself whether your answer makes sense.
10. Write your answer statement.
Show any or all your work! Do not leave a blank page!

My Strategies:
Answer and Solution:
Find all missing angles.

Apply Sine law to find c.

\[
\frac{a}{\sin A} = \frac{c}{\sin C}
\]

\[
\frac{353}{\sin 30} = \frac{c}{\sin 130}
\]

\[
c = 541 \text{ m}
\]

Apply trig to find \( \overline{AD} \) (height of Space Needle).

\[
\sin 20 = \frac{\overline{AD}}{541}
\]

\[
\overline{AD} = 185 \text{ m}
\]

☑ Yes, his estimation is correct.
☐ No, his estimation is not correct.

Reason: The clinometer measures yield a height of 185 m for the Space Needle. This falls within Phil’s estimation of 182 m to 188 m.